# MECA 321 – Static Failure Analysis of a 6-DOF Robotic Arm

Mechanics of Materials (Hand-Calculation Approach)


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## 1 Design Requirements and Assumptions

### 1.1 Design Requirements

- Payload: 50 kg
- Reach: 2 m from base axis to payload point
- Minimum factor of safety: FoS $\geq 2$
- Use industrial standard profiles (IPE family)

### 1.2 Assumptions

- Linear elastic material behavior.
- Static loading conditions (no dynamics, impacts, or fatigue).
- Worst-case pose is fully horizontal reach (max gravitational moment arm).
- Conservative structural idealization: equivalent straight cantilever of length $L = 2.0$ m.
- Beam self-weight included as distributed load using selected profile mass-per-meter.
- CAD non-beam masses (motors/gearboxes/housings/bearings/gripper mechanisms) are excluded from baseline but bounded via a conservative sensitivity check (added tip mass).

# 2 Conceptual Design

The robotic arm is a serial industrial manipulator with six revolute joints (6R). The joints represent base yaw, shoulder pitch, elbow pitch, and a 3-DOF wrist (pitch/yaw/roll), consistent with common industrial robot architectures.

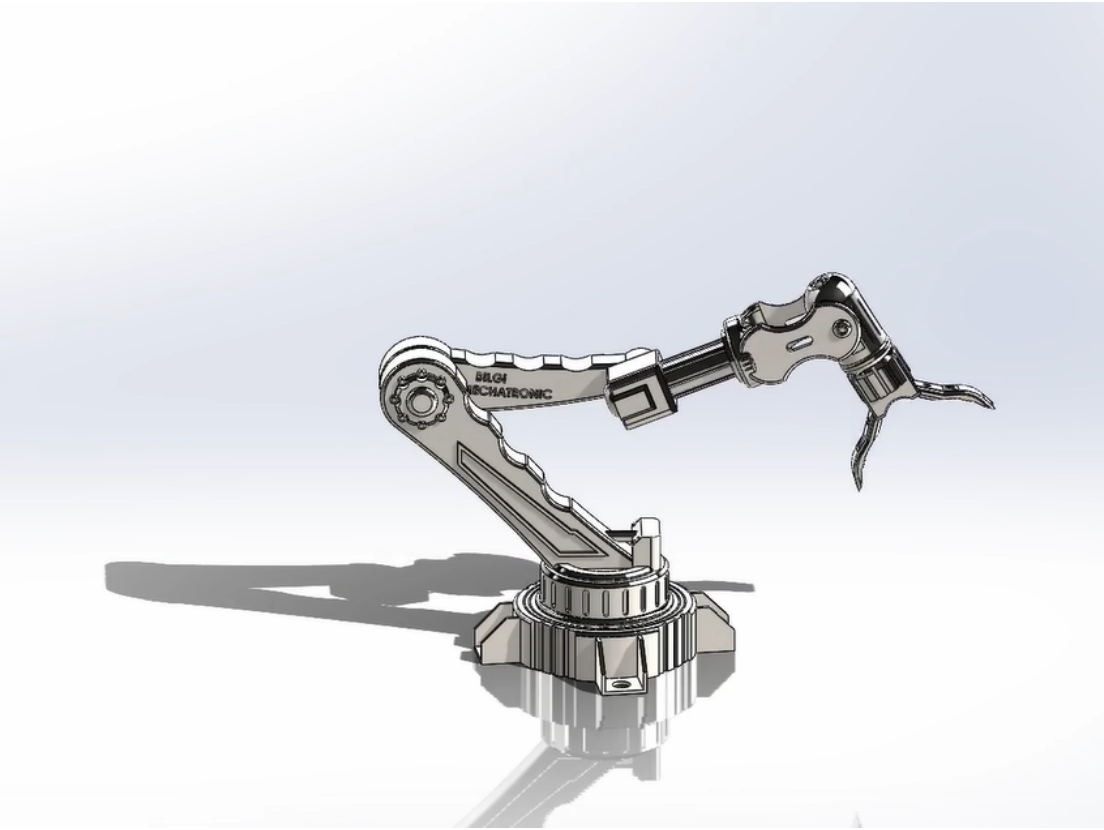

Figure 1: Overall configuration of the robotic arm (SolidWorks render).

# 3 CAD Design (SolidWorks)

A 3D CAD assembly was created in SolidWorks and scaled to satisfy the 2 m reach requirement.

## 3.1 Link lengths and scaling

CAD link lengths (original model units):

$$\text{Lower arm link} = 162, \quad \text{Upper arm link} = 155, \quad \text{Shoulder+yoke} = 145,$$
$$\text{Wrist module} = 50, \quad \text{Gripper actuator} = 25, \quad \text{Gripper} = 50$$

(All CAD link lengths above are in mm.)

Total CAD reach:

$$L_{\text{CAD}} = 162 + 155 + 145 + 50 + 25 + 50 = 587$$

Required reach:

$$L_{\text{req}} = 2.0\,\text{m} = 2000\,\text{mm}$$

Scale factor:

$$k = \frac{2000}{587} = 3.406$$

Scaled link lengths (meters):

$$L_1 = 0.552,\ L_2 = 0.528,\ L_3 = 0.494,\ L_4 = 0.170,\ L_5 = 0.085,\ L_6 = 0.170, \qquad \sum L_i \approx 2.0\,\mathrm{m}$$

### 3.2 Key hardware dimensions (concept-stage)

- Pin diameter (shoulder and elbow, bounding): $d_p = 16\,\mathrm{mm}$
- Lug thickness (bounding): $t = 10\,\mathrm{mm}$
- Base bolt pattern (bounding): 4 bolts on bolt-circle radius $r_b = 100\,\mathrm{mm}$

## 4 Material and Cross-Section Selection

### 4.1 Material selection

Structural steel S355 is selected for the beam links (properties per EN 10025-2)[1]. Material properties:

$$E = 210\,\mathrm{GPa}, \qquad \sigma_y = 355\,\mathrm{MPa}, \qquad \nu = 0.30, \qquad \rho \approx 7850\,\mathrm{kg/m^3}$$

Allowable normal stress for FoS = 2:

$$\sigma_{\text{allow}} = \frac{\sigma_y}{2} = 177.5\,\mathrm{MPa}$$

### 4.2 Material selection (required format)

Table 1: Material selection and key properties.

| **Component** | **Material** | $E$ **(GPa)** | $\sigma_y$ **(MPa)** | **Density (kg/m$^3$)** |
|---|---|---|---|---|
| Beam links (all iterations) | S355 | 210 | 355 | 7850 |
| Pins (bounding check) | Steel (S355 bound) | 210 | 355 | 7850 |
| Base bolts (M12, class 8.8) | ISO 8.8 | 210 | 640 | 7850 |

Bolt property class reference: ISO 898-1 [6].

### 4.3 Cross-section property data used (IPE/IPEA/SHS)

Section properties used in the loading and stress calculations are taken from standard section tables and online databases consistent with EN-defined profiles.[2, 3, 4, 5] Throughout this report, bending is about the profile strong axis; we denote this as the $x$-axis (reported using $I_x$ and $W_{el,x}$).

The SHS properties correspond to a hot-finished square hollow section standardized in EN 10210.[5]

Table 2: Cross-section properties used across Iterations 1–4.

| **Section** | **Mass (kg/m)** | $A$ **(mm²)** | $W_{el,x}$ **(mm³)** | $I_x$ **(mm⁴)** |
|---|---|---|---|---|
| IPE 120 | 10.4 | 1320 | 52,950 | 3,177,000 |
| IPE 100 | 8.1 | 1030 | 34,200 | 1,710,000 |
| IPE 80 | 6.0 | 760 | 20,030 | 801,300 |
| IPEA 120 | 8.7 | 1100 | 43,760 | 2,573,000 |
| SHS 50×50×2 | 3.01 | 384 | 5908 | 147,712 |

Table 3: Member profile assignment by iteration (equivalent straight-cantilever model).

| **Link group (station range)** | **Iteration 1** | **Iteration 2** | **Iteration 3** | **Iteration 4** |
|---|---|---|---|---|
| Link 1 (J1→J2) | IPE 120 | IPE 120 | IPE 120 | SHS 50×50×2 |
| Link 2 (J2→J3) | IPE 120 | IPE 120 | IPE 100 | SHS 50×50×2 |
| Links 3–6 (J3→Tip) | IPE 120 | IPEA 120 | IPE 80 | SHS 50×50×2 |

# 5 Structural Analysis for Static Failure

## 5.1 Loads and worst-case configuration

Payload (tip load):

$$P = mg = 50(9.81) = 490.5\,\text{N}$$

Worst case is the fully horizontal reach, modeled as a straight cantilever of length $L = 2.0\,\text{m}$.

Baseline self-weight (Iteration 1, all IPE120):

$$w = (10.4)(9.81) = 102\,\text{N/m}$$

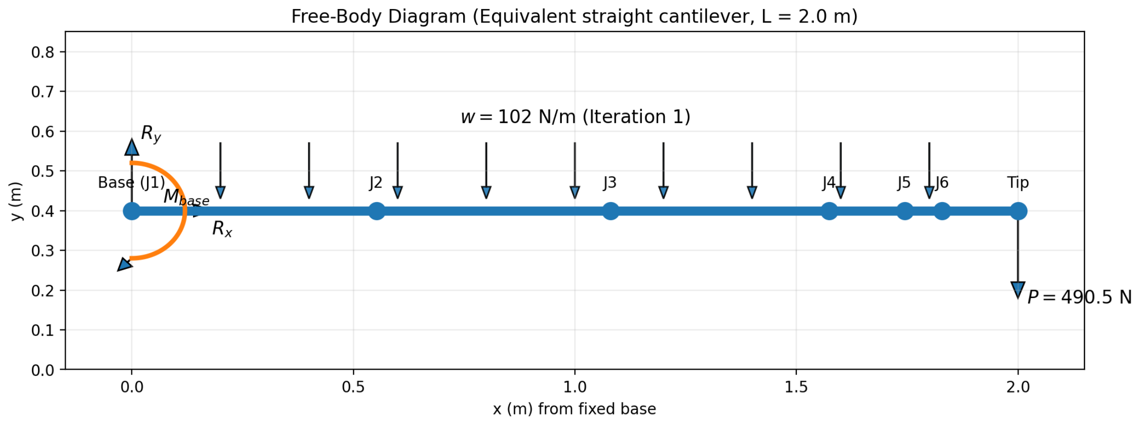


Figure 2: Free-body diagram of the equivalent straight cantilever model ($L = 2.0\,\text{m}$) including payload $P$ and self-weight $w$.

## 5.2 Example equation formats (per brief)

The mechanics-of-materials relations used below follow standard beam theory and are consistent with course texts such as Beer et al. [7].

$$\sigma = \frac{Mc}{I} = \frac{M}{W} \tag{1}$$

$$\delta_P = \frac{PL^3}{3EI} \tag{2}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \tag{3}$$

## 5.3 Shear force and bending moment (payload + UDL)

For a cantilever with end load $P$ and uniform load $w$:

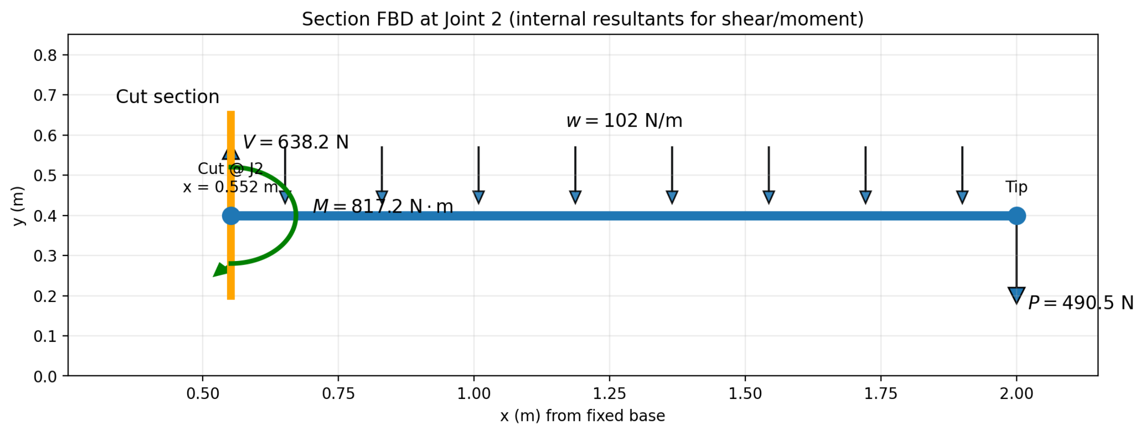


Figure 3: Section cut at Joint 2 showing internal shear $V$ and bending moment $M$ used for shear/moment/deflection calculations.

$$V(x) = P + w(L - x), \qquad 0 \le x \le L \tag{4}$$

$$M(x) = P(L - x) + \frac{w}{2}(L - x)^2, \qquad 0 \le x \le L \tag{5}$$

At the base ($x = 0$):

$$V_{\text{max}} = P + wL = 490.5 + 102(2) = 694.5\,\text{N}$$

$$M_{\text{max}} = PL + \frac{wL^2}{2} = 490.5(2) + \frac{102(2^2)}{2} \approx 1185\,\text{N}\cdot\text{m}$$

## 5.4 Bending stress check (Iteration 1 baseline: IPE120 everywhere)

Convert $M_{\text{max}}$:

$$M_{\text{max}} = 1185\,\text{N}\cdot\text{m} = 1.185 \times 10^6\ \text{N}\cdot\text{mm}$$

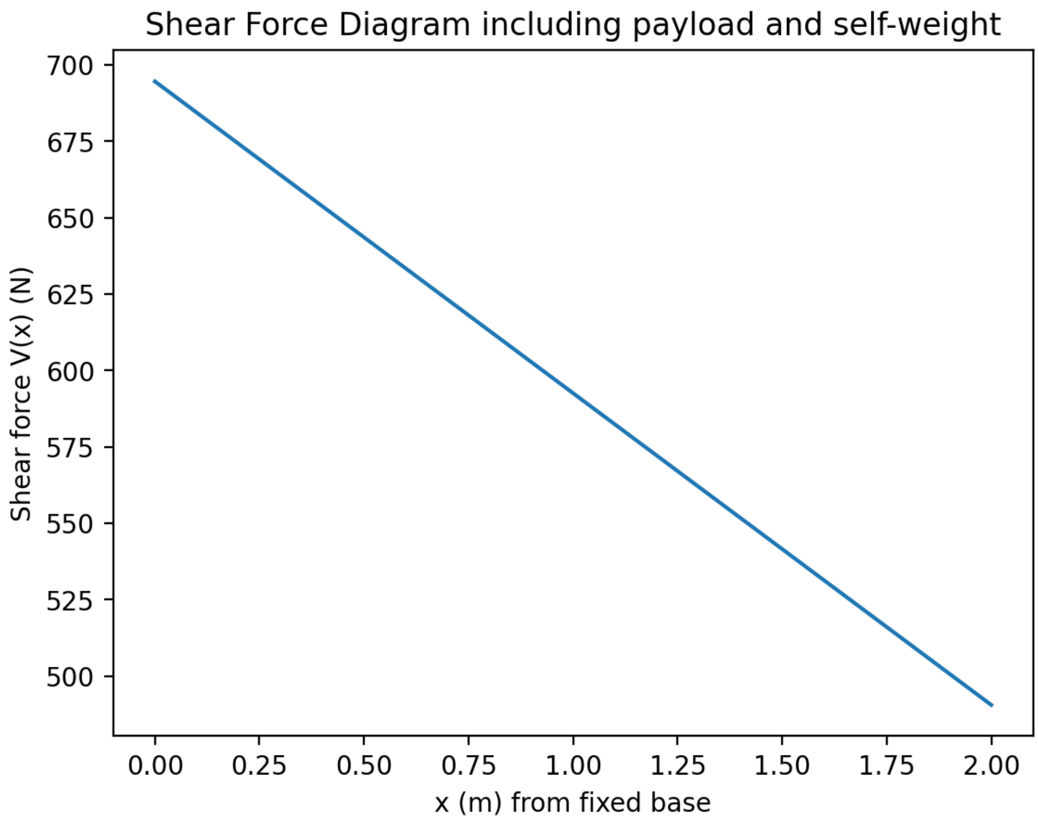


Figure 4: Shear force diagram including payload and self-weight (Iteration 1 baseline).

With IPE120, $W_{el,x} = 52.95\,\text{cm}^3 = 52{,}950\,\text{mm}^3$:

$$\sigma_{\max} = \frac{M_{\max}}{W_x} = \frac{1.185 \times 10^6}{52950} = 22.38\,\text{MPa}$$

$$\text{FoS}_{\text{bend}} = \frac{\sigma_y}{\sigma_{\max}} = \frac{355}{22.38} = 15.9 \geq 2$$

### 5.5 Shear stress check (conservative web-average, IPE120)

Using web thickness $s = 4.4\,\text{mm}$, height $h = 120\,\text{mm}$, flange thickness $t = 6.3\,\text{mm}$:

$$A_{\text{web}} \approx s(h - 2t) = 4.4(120 - 2(6.3)) = 472.6\,\text{mm}^2$$

$$\tau_{\text{avg}} \approx \frac{V_{\max}}{A_{\text{web}}} = \frac{694.5}{472.6} = 1.47\,\text{MPa}$$

Allowable shear (von Mises estimate):

$$\tau_{\text{allow}} \approx \frac{0.577\sigma_y}{\text{FoS}}, \qquad \text{FoS} = 2 \Rightarrow \tau_{\text{allow}} = \frac{0.577(355)}{2} = 102.4\,\text{MPa}$$

Thus shear is non-governing.

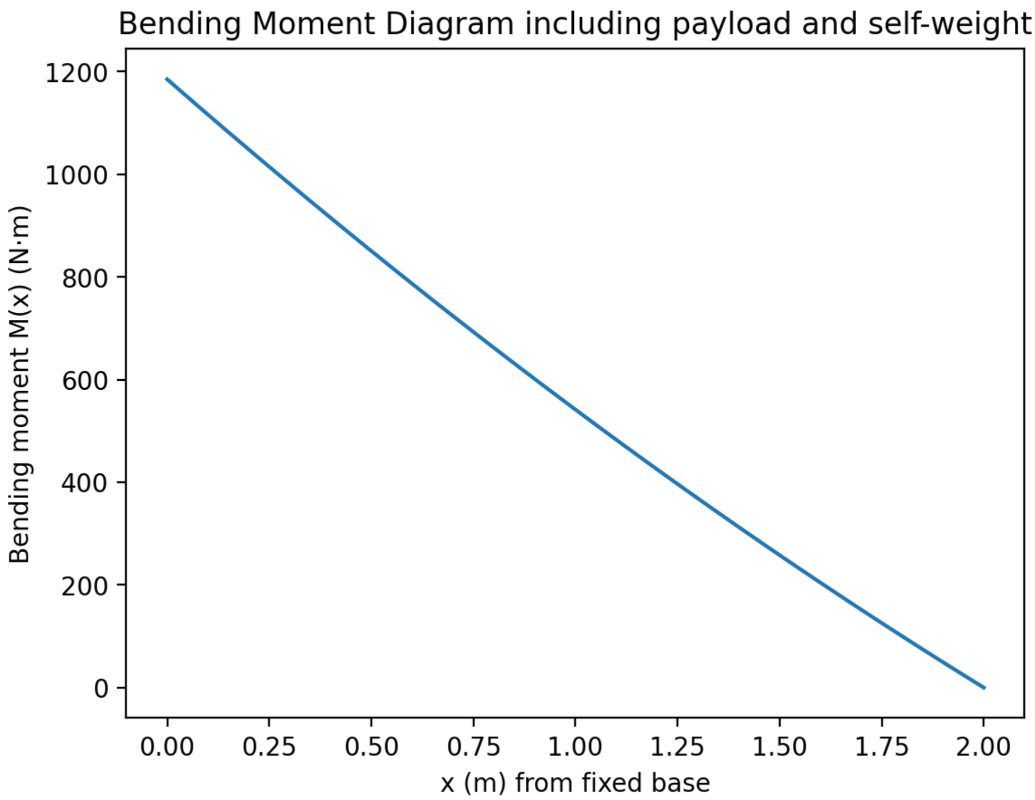


Figure 5: Bending moment diagram including payload and self-weight (Iteration 1 baseline).

## 5.6 Deflection analysis (Iteration 1 baseline, constant $EI$)

IPE120 $I_x = 317.7\,\text{cm}^4 = 3.177 \times 10^{-6}$ m$^4$.

$$\delta_P = \frac{PL^3}{3EI} = \frac{490.5(2^3)}{3(210 \times 10^9)(3.177 \times 10^{-6})} = 1.96\,\text{mm}$$

$$\delta_w = \frac{wL^4}{8EI} = \frac{102(2^4)}{8(210 \times 10^9)(3.177 \times 10^{-6})} = 0.31\,\text{mm}$$

$$\delta_{\text{tot}} \approx 2.27\,\text{mm}$$

## 5.7 Other failure modes (bounding checks)

### 5.7.1 Axial stress bound

Conservative bound: $N \approx P = 490.5\,\text{N}$. Using IPE120 area $A \approx 1320\,\text{mm}^2$:

$$\sigma_a = \frac{N}{A} = \frac{490.5}{1320} = 0.372\,\text{MPa} \ll \sigma_{\text{allow}}$$

### 5.7.2 Torsion bound (wrist shaft)

Assume eccentricity $e = 0.10\,\text{m}$:

$$T = Pe = 490.5(0.10) = 49.05\,\text{N} \cdot \text{m} = 49{,}050\,\text{N} \cdot \text{mm}$$

Solid shaft (bounding) with $d = 16\,\text{mm}$:

$$\tau_{\max} = \frac{16T}{\pi d^3} = \frac{16(49050)}{\pi(16^3)} \approx 61\,\text{MPa}$$

Allowable shear (von Mises estimate) $\tau_{\text{allow}} \approx 102.4\,\text{MPa}$, so torsion is safe.

### 5.7.3 Euler buckling (bounding)

A conservative compression case is checked with $N = 694.5\,\text{N}$, $L_c = 2.0\,\text{m}$, and cantilever effective length factor $K = 2$. Using minor-axis inertia $I_{\min} = 27.7\,\text{cm}^4 = 2.77 \times 10^5\ \text{mm}^4$ and $E = 210{,}000\,\text{MPa}$:

$$P_{cr} = \frac{\pi^2 E I_{\min}}{(KL_c)^2} \approx 3.59 \times 10^4\,\text{N}$$

$$\text{FoS}_{\text{buckling}} = \frac{P_{cr}}{N} \approx \frac{35900}{694.5} \approx 52$$

Buckling is non-governing for the horizontal worst-case bend-dominated pose.

## 5.8 Sensitivity check (excluded masses)

To bound the effect of unmodeled concentrated masses (motors/gearboxes/housings), an additional tip mass of $m_{add} = 10\,\text{kg}$ is conservatively added:

$$P_{add} = m_{add} g = 10(9.81) = 98.1\,\text{N}$$

$$\Delta M = P_{add} L = 98.1(2.0) = 196.2\,\text{N}\cdot\text{m}$$

Thus $M \approx 1185 + 196.2 = 1381.2\,\text{N}\cdot\text{m}$ and for IPE120:

$$\sigma \approx \frac{1.3812 \times 10^6}{52950} = 26.1\,\text{MPa} \Rightarrow \text{FoS} \approx \frac{355}{26.1} = 13.6$$

The design remains safe with significant margin.

# 6 Design Iteration and Optimization

## 6.1 Iteration strategy and rationale

To satisfy the rubric requirement to *iterate and minimize weight*, the design is optimized in stages. Because the bending moment is maximum at the base and decreases toward the tip, the most efficient approach is to keep the base section robust while reducing distal mass:

- **Iteration 1 (baseline):** one standard IPE profile everywhere to establish the FBDs, shear/moment/deflection, and required section modulus.
- **Iteration 2:** reduce mass in distal links (lower-moment region) while keeping the base link unchanged.
- **Iteration 3 (final):** apply a three-step taper (IPE120 → IPE100 → IPE80) to maximize mass/cost reduction while keeping deflection small.

- **Iteration 4 (min. cost target):** a strength-limited profile sized so that FoS $\approx 2$; included to show the practical lower bound and the stiffness trade-off.

The baseline design (IPE120 everywhere) is overly conservative in stress and deflection. Therefore, a tapered selection of standard IPE profiles is applied to reduce mass while maintaining FoS $\geq 2$.

## 6.2 Iteration definitions

- **Iteration 1:** IPE120 for all links ($L = 2.0\,\mathrm{m}$ total).
- **Iteration 2:** IPE120 for proximal links; IPEA120 for distal links (mass reduction).
- **Iteration 3 (final):** IPE120 on Link 1, IPE100 on Link 2, IPE80 on Links 3–6.

## 6.3 Iteration 3: updated moments at key stations (payload + updated self-weight)

Using centroid-based summation of distributed loads for Iteration 3, the total bending moments at key stations are:

$$M_{T,1} \approx 1114\,\mathrm{N\cdot m}, \quad M_{T,2} \approx 774.7\,\mathrm{N\cdot m}, \quad M_{T,3} \approx 476.1\,\mathrm{N\cdot m}$$

Corresponding bending stresses (using Eq. (1)):

$$\sigma_1 = \frac{1.114\times 10^6}{52950} = 21.0\,\mathrm{MPa}, \quad \sigma_2 = \frac{774700}{34200} = 22.6\,\mathrm{MPa}, \quad \sigma_3 = \frac{476100}{20030} = 23.8\,\mathrm{MPa}$$

All satisfy FoS $\geq 2$.

**Optional: dynamic amplification and actuator sizing note.** All results above are for quasi-static loading (gravity only, no impacts). If dynamic effects are expected (rapid acceleration/deceleration, stop/start shocks, payload swing), a conservative dynamic load factor (DLF) can be applied directly because the system is linear in the loads:

$$V_{\mathrm{dyn}}(x) = \mathrm{DLF}\,V(x), \qquad M_{\mathrm{dyn}}(x) = \mathrm{DLF}\,M(x).$$

For example, with DLF = 1.5, the Iteration 3 shoulder station (J2) moment becomes

$$M_{\mathrm{dyn}}(J2) \approx 1.5 \times 774.8\,\mathrm{N\,m} = 1162\,\mathrm{N\,m}.$$

Under the same simplified planar assumption, this bending moment is the governing equivalent joint torque about the out-of-plane axis for the horizontal pose, and it provides a clear sizing target for the actuator + gearbox at that joint.

### 6.3.1 Joint-by-joint shear and bending moment (all iterations)

For actuator sizing and joint hardware checks, the internal *shear force* $V$ and *bending moment* $M$ are reported at each revolute joint location for **all four iterations** under the same worst-case pose: the arm is fully horizontal, carrying the payload $P = 490.5\,\mathrm{N}$ at the tool center (2.0 m from the base).

Table 4: Joint station locations along the equivalent straight cantilever axis (Figure 2).

| **Station** | $x$ **(m)** |
|---|---|
| J1 (Base) | 0.000 |
| J2 | 0.552 |
| J3 | 1.080 |
| J4 | 1.574 |
| J5 | 1.744 |
| J6 | 1.829 |
| Tip (tool center) | 2.000 |

**Joint station locations.** The multi-link arm is mapped to an equivalent straight cantilever (Figure 2) using the measured link lengths, giving the cut locations in Table 4.

**Governing equations (consistent with Figures 4–5).** With downward distributed self-weight $w(s)$ along the span, the internal resultants at a cut located at $x$ are computed from the loads to the *right* of the cut:

$$V(x) = P + \int_x^L w(s)\, ds, \qquad M(x) = P(L - x) + \int_x^L w(s)\,(s - x)\, ds,$$

reported as positive magnitudes (i.e., the internal actions required to balance the gravity loads). For a *uniform* $w$, this reduces to the closed form used to generate the shear/moment plots:

$$V(x) = P + w(L - x), \qquad M(x) = P(L - x) + \frac{w}{2}(L - x)^2.$$

This matches the joint-2 cut shown in Figure 3: at $x = 0.552\,\mathrm{m}$ with $w = 102\,\mathrm{N/m}$ and $L = 2\,\mathrm{m}$,

$$V(0.552) = 490.5 + 102(2 - 0.552) = 638.2\,\mathrm{N},$$
$$M(0.552) = 490.5(2 - 0.552) + \frac{102}{2}(2 - 0.552)^2 = 817.2\,\mathrm{N\,m}.$$

For *piecewise-constant* distributed loads (Iterations 2–3), each segment $i$ has constant $w_i$ over $[a_i, b_i]$. Define $s_0 = \max(x, a_i)$ and $s_1 = \min(L, b_i)$. If $s_1 > s_0$, the segment contributes

$$\Delta V_i = w_i(s_1 - s_0), \qquad \Delta M_i = \frac{w_i}{2}\Big[(s_1 - x)^2 - (s_0 - x)^2\Big],$$

and $V(x) = P + \sum_i \Delta V_i$, $M(x) = P(L - x) + \sum_i \Delta M_i$.

**Iteration 1 (IPE120 everywhere).** Self-weight is uniform: $w = (10.4)(9.81) \approx 102\,\mathrm{N/m}$ over $L = 2\,\mathrm{m}$.

**Station-wise results (shear and equivalent bending torque).** The internal shear $V$ and bending moment $M$ at each station are taken from the table above. In the static horizontal pose, $M(x)$ is also a direct proxy for the required holding torque about the out-of-plane joint axis at that station.

Table 5: Joint-by-joint internal shear and bending moment for Iteration 1 (uniform IPE120, $w = 102\,\mathrm{N/m}$, $P = 490.5\,\mathrm{N}$).

| **Station** | $x$ **(m)** | $V$ **(N)** | $M$ **(N·m)** |
|---|---|---|---|
| J1 (Base) | 0.000 | 694.5 | 1185.0 |
| J2 | 0.552 | 638.2 | 817.2 |
| J3 | 1.080 | 584.3 | 494.4 |
| J4 | 1.574 | 534.0 | 218.2 |
| J5 | 1.744 | 516.6 | 128.9 |
| J6 | 1.829 | 507.9 | 85.4 |
| Tip | 2.000 | 490.5 | 0.0 |

- J1 (Base) ($x = 0.000\,\mathrm{m}$): $V = 694.5\,\mathrm{N}$, $M = 1185.0\,\mathrm{N\cdot m}$.
- J2 ($x = 0.552\,\mathrm{m}$): $V = 638.2\,\mathrm{N}$, $M = 817.2\,\mathrm{N\cdot m}$.
- J3 ($x = 1.080\,\mathrm{m}$): $V = 584.3\,\mathrm{N}$, $M = 494.4\,\mathrm{N\cdot m}$.
- J4 ($x = 1.574\,\mathrm{m}$): $V = 534.0\,\mathrm{N}$, $M = 218.2\,\mathrm{N\cdot m}$.
- J5 ($x = 1.744\,\mathrm{m}$): $V = 516.6\,\mathrm{N}$, $M = 128.9\,\mathrm{N\cdot m}$.
- J6 ($x = 1.829\,\mathrm{m}$): $V = 507.9\,\mathrm{N}$, $M = 85.4\,\mathrm{N\cdot m}$.
- Tip ($x = 2.000\,\mathrm{m}$): $V = 490.5\,\mathrm{N}$, $M = 0.0\,\mathrm{N\cdot m}$.

**Iteration 2 (IPE120 Links 1–2, IPEA120 Links 3–6).** Links 1–2 (up to $x = 1.08\,\mathrm{m}$) use IPE120: $w_{1-2} = 102\,\mathrm{N/m}$. Links 3–6 (from $x = 1.08\,\mathrm{m}$ to the tip) use IPEA120 with mass 8.7 kg/m[4], giving $w_{3-6} = (8.7)(9.81) = 85.3\,\mathrm{N/m}$.

Table 6: Joint-by-joint internal shear and bending moment for Iteration 2 (IPE120 on Links 1–2, IPEA120 on Links 3–6, $P = 490.5\,\mathrm{N}$).

| **Station** | $x$ **(m)** | $V$ **(N)** | $M$ **(N·m)** |
|---|---|---|---|
| J1 (Base) | 0.000 | 679.2 | 1161.4 |
| J2 | 0.552 | 622.9 | 802.0 |
| J3 | 1.080 | 569.0 | 487.4 |
| J4 | 1.574 | 526.9 | 216.7 |
| J5 | 1.744 | 512.3 | 128.4 |
| J6 | 1.829 | 505.1 | 85.1 |
| Tip | 2.000 | 490.5 | 0.0 |

**Station-wise results (shear and equivalent bending torque).** The internal shear $V$ and bending moment $M$ at each station are taken from the table above. In the static horizontal pose, $M(x)$ is also a direct proxy for the required holding torque about the out-of-plane joint axis at that station.

- J1 (Base) ($x = 0.000\,\mathrm{m}$): $V = 679.2\,\mathrm{N}$, $M = 1161.4\,\mathrm{N\cdot m}$.
- J2 ($x = 0.552\,\mathrm{m}$): $V = 622.9\,\mathrm{N}$, $M = 802.0\,\mathrm{N\cdot m}$.

- J3 ($x = 1.080\,\text{m}$): $V = 569.0\,\text{N}$, $M = 487.4\,\text{N}\cdot\text{m}$.
- J4 ($x = 1.574\,\text{m}$): $V = 526.9\,\text{N}$, $M = 216.7\,\text{N}\cdot\text{m}$.
- J5 ($x = 1.744\,\text{m}$): $V = 512.3\,\text{N}$, $M = 128.4\,\text{N}\cdot\text{m}$.
- J6 ($x = 1.829\,\text{m}$): $V = 505.1\,\text{N}$, $M = 85.1\,\text{N}\cdot\text{m}$.
- Tip ($x = 2.000\,\text{m}$): $V = 490.5\,\text{N}$, $M = 0.0\,\text{N}\cdot\text{m}$.

**Iteration 2 strength and deflection check.** The peak bending moment occurs at the base: $M_{base} = 1161.4\,\text{N}\cdot\text{m}$ (Table 6). With IPE120 $W_{el,x} = 52{,}950\,\text{mm}^3$ (Table 2),

$$\sigma_{base} = \frac{1.161 \times 10^6}{52950} = 21.9\,\text{MPa} \quad \Rightarrow \quad \text{FoS}_{bend} \approx \frac{355}{21.9} \approx 16.2.$$

At the start of Links 3–6 (IPEA120), the largest moment is at $x = 1.08\,\text{m}$ with $M = 487.4\,\text{N}\cdot\text{m}$, giving $\sigma \approx (0.487 \times 10^6)/43760 \approx 11.1\,\text{MPa}$, so bending is base-governed in this iteration. A unit-load integration using the piecewise stiffness (IPE120 for $0 \leq x < 1.08\,\text{m}$ and IPEA120 for $1.08\,\text{m} \leq x \leq 2.0\,\text{m}$) and the same piecewise $w(x)$ gives the tip deflection:

$$\delta_2 \approx 2.27\,\text{mm}.$$

**Iteration 3 (IPE120 Link 1, IPE100 Link 2, IPE80 Links 3–6).**

$$w_1 = (10.4)(9.81) = 102.0\,\text{N/m}, \quad w_2 = (8.1)(9.81) = 79.5\,\text{N/m}, \quad w_{3-6} = (6.0)(9.81) = 58.9\,\text{N/m}.$$

Table 7: Joint-by-joint internal shear and bending moment for Iteration 3 (IPE120 on Link 1, IPE100 on Link 2, IPE80 on Links 3–6, $P = 490.5\,\text{N}$).

| **Station** | $x$ **(m)** | $V$ **(N)** | $M$ **(N·m)** |
|---|---|---|---|
| J1 (Base) | 0.000 | 642.9 | 1114.2 |
| J2 | 0.552 | 586.6 | 774.8 |
| J3 | 1.080 | 544.7 | 476.2 |
| J4 | 1.574 | 515.6 | 214.3 |
| J5 | 1.744 | 505.6 | 127.5 |
| J6 | 1.829 | 500.6 | 84.7 |
| Tip | 2.000 | 490.5 | 0.0 |

**Station-wise results (shear and equivalent bending torque).** The internal shear $V$ and bending moment $M$ at each station are taken from the table above. In the static horizontal pose, $M(x)$ is also a direct proxy for the required holding torque about the out-of-plane joint axis at that station.

- J1 (Base) ($x = 0.000\,\text{m}$): $V = 642.9\,\text{N}$, $M = 1114.2\,\text{N}\cdot\text{m}$.
- J2 ($x = 0.552\,\text{m}$): $V = 586.6\,\text{N}$, $M = 774.8\,\text{N}\cdot\text{m}$.
- J3 ($x = 1.080\,\text{m}$): $V = 544.7\,\text{N}$, $M = 476.2\,\text{N}\cdot\text{m}$.
- J4 ($x = 1.574\,\text{m}$): $V = 515.6\,\text{N}$, $M = 214.3\,\text{N}\cdot\text{m}$.

- J5 ($x = 1.744$ m): $V = 505.6$ N, $M = 127.5$ N · m.
- J6 ($x = 1.829$ m): $V = 500.6$ N, $M = 84.7$ N · m.
- Tip ($x = 2.000$ m): $V = 490.5$ N, $M = 0.0$ N · m.

**Iteration 4 (SHS 50 mm×50 mm×2 mm everywhere).** The hollow section has mass per length $m' = \rho A = (7850)(3.84 \times 10^{-4}) \approx 3.01$ kg/m, hence $w = m'g \approx 29.55$ N/m uniformly over the 2 m reach.

Table 8: Joint-by-joint internal shear and bending moment for Iteration 4 (SHS 50 mm×50 mm×2 mm, $P = 490.5$ N).

| **Station** | $x$ **(m)** | $V$ **(N)** | $M$ **(N·m)** |
|---|---|---|---|
| J1 (Base) | 0.000 | 549.6 | 1040.1 |
| J2 | 0.552 | 533.3 | 741.2 |
| J3 | 1.080 | 517.7 | 463.8 |
| J4 | 1.574 | 503.1 | 211.6 |
| J5 | 1.744 | 498.1 | 126.5 |
| J6 | 1.829 | 495.6 | 84.3 |
| Tip | 2.000 | 490.5 | 0.0 |

**Station-wise results (shear and equivalent bending torque).** The internal shear $V$ and bending moment $M$ at each station are taken from the table above. In the static horizontal pose, $M(x)$ is also a direct proxy for the required holding torque about the out-of-plane joint axis at that station.

- J1 (Base) ($x = 0.000$ m): $V = 549.6$ N, $M = 1040.1$ N · m.
- J2 ($x = 0.552$ m): $V = 533.3$ N, $M = 741.2$ N · m.
- J3 ($x = 1.080$ m): $V = 517.7$ N, $M = 463.8$ N · m.
- J4 ($x = 1.574$ m): $V = 503.1$ N, $M = 211.6$ N · m.
- J5 ($x = 1.744$ m): $V = 498.1$ N, $M = 126.5$ N · m.
- J6 ($x = 1.829$ m): $V = 495.6$ N, $M = 84.3$ N · m.
- Tip ($x = 2.000$ m): $V = 490.5$ N, $M = 0.0$ N · m.

These values are gravity-induced. In dynamic operation, joints also experience motor torques and pose-dependent inertial effects; the tables above provide a conservative static baseline for sizing.

### 6.3.2 Bounding joint pin shear/bearing check (concept-stage)

Using the concept-stage joint pin diameter $d_p = 16$ mm and lug thickness $t = 10$ mm (double shear, two lugs), the pin shear stress and lug bearing pressure are:

$$\tau_p = \frac{V}{2A_p}, \qquad A_p = \frac{\pi d_p^2}{4}, \qquad p_b = \frac{V}{2td_p}.$$

Table 9: Pin shear stress and lug bearing pressure at the main joints (final design: Iteration 3 loads from Table 7).

| **Joint** | $V$ **(N)** | $\tau_p$ **(MPa)** | $p_b$ **(MPa)** |
|---|---|---|---|
| Joint 2 | 586.6 | 1.459 | 1.833 |
| Joint 3 | 544.7 | 1.354 | 1.702 |
| Joint 4 | 515.6 | 1.282 | 1.611 |
| Joint 5 | 505.6 | 1.257 | 1.580 |
| Joint 6 | 500.6 | 1.245 | 1.564 |

These stresses are far below the S355 yield strength; therefore, under the static-gravity load case, joint pin shear/bearing is non-governing. Dynamic loads, impact, and fatigue should be considered in a detailed design phase.

### 6.3.3 Base bolt-group check (overturning moment + shear, bounding)

The base is assumed to be fixed to the ground via a symmetric 4-bolt pattern on a bolt-circle radius $r_b = 100\,\mathrm{mm}$. Using the Iteration 3 base shear and moment from Table 7, a simple elastic bolt-group estimate gives:

$$F_{t,\max} \approx \frac{M}{n r_b}, \qquad F_v \approx \frac{V}{n},$$

with $n = 4$. For $M = 1114.2\,\mathrm{N}\cdot\mathrm{m}$ and $V = 642.9\,\mathrm{N}$:

$$F_{t,\max} \approx 2786\,\mathrm{N}\ \ \text{(per bolt)}, \qquad F_v \approx 161\,\mathrm{N}\ \ \text{(per bolt)}.$$

For an M12 bolt (tensile stress area $A_t \approx 84.3\,\mathrm{mm}^2$), the corresponding stresses are:

$$\sigma_b \approx \frac{F_{t,\max}}{A_t} = 33.0\,\mathrm{MPa}, \qquad \tau_b \approx \frac{F_v}{A_t} = 1.91\,\mathrm{MPa},$$

and the combined von Mises stress $\sigma_{vm} \approx \sqrt{\sigma_b^2 + 3\tau_b^2} \approx 33.2\,\mathrm{MPa}$, which is well below a conservative allowable for class 8.8 bolts (e.g., $\sigma_y/2 \approx 320\,\mathrm{MPa}$).

## 6.4 Iteration 3 deflection (piecewise $EI$)

A piecewise stiffness estimate is performed using the unit-load method:

$$\delta(L) = \int_0^L \frac{M(x)\,m(x)}{EI(x)}\,dx$$

where $m(x) = L - x$ is the unit-load bending moment at the tip. With piecewise $I(x)$ for IPE120/IPE100/IPE80 segments and including payload plus self-weight, the predicted tip deflection is:

$$\delta_3 \approx 3.25\,\mathrm{mm}$$

This remains small relative to the 2 m reach and does not govern static failure.

### 6.5 Iteration 4: Lowest-cost strength-limited design (target FoS $\approx$ 2)

Iteration 4 explores the lowest-cost option that still satisfies the project requirement FoS $\geq 2$. Instead of selecting an IPE section that produces a large safety margin, a standard **square hollow section (SHS)** is sized so that the base bending stress approaches $\sigma_{\text{allow}} = \sigma_y/2$. Structural hollow sections are standardized in EN 10210 [5].

A practical standard size is **SHS 50 mm×50 mm×2 mm** with:

$$A = 384\,\text{mm}^2, \quad I = 147{,}712\,\text{mm}^4, \quad W = 5908\,\text{mm}^3.$$

Table 10: SHS 50 mm×50 mm×2 mm section-property calculation (used in Iteration 4).

| **Property** | **Formula (square tube)** | **Value** |
|---|---|---|
| Outer width | $b$ | 50 |
| Wall thickness | $t$ | 2 |
| Area | $A = b^2 - (b-2t)^2$ | 384 |
| Second moment | $I_x = \dfrac{b^4 - (b-2t)^4}{12}$ | 147,712 |
| Section modulus | $W_x = \dfrac{I_x}{b/2}$ | 5908 |

Using the same cantilever model with $L = 2\,\text{m}$, payload $P = 490.5\,\text{N}$, and self-weight $w = m'g$:

$$M_4(0) = PL + \frac{wL^2}{2} = 1040.1\,\text{N m}, \qquad \sigma_{4,\max} = \frac{M_4(0)}{W} \approx 176.04\,\text{MPa}, \qquad n_4 \approx 2.02.$$

This meets the minimum safety requirement. However, the stiffness penalty is significant:

$$\delta_4 = \frac{PL^3}{3EI} + \frac{wL^4}{8EI} \approx 44.1\,\text{mm}.$$

Therefore, Iteration 4 is the *lowest-cost strength-compliant* case, but it is not selected as the final design due to its much larger deflection (precision/stiffness concerns).

Table 11: Design iteration and optimization summary.

| **Design** | **Mass (kg)** | **Max Stress (MPa)** | **FOS** |
|---|---|---|---|
| Iteration 1: IPE120 everywhere | 20.80 | 22.38 | 15.9 |
| Iteration 2: IPE120 + IPEA120 | 19.24 | 21.90 | 16.2 |
| Iteration 3 (recommended): IPE120 + IPE100 + IPE80 | 15.54 | 23.80 | 14.9 |
| Iteration 4 (min. cost): SHS 50×50×2 | 6.02 | 176.04 | 2.02 |

# 7 Cost Analysis

We assumed the value (40 TL/kg) represents a rounded, conservative estimate of the raw material cost for hot-rolled structural steel sections (such as IPE beams) in the Turkish market as of late 2025. While actual supplier prices for finished IPE profiles fluctuate based on factors like order volume, exact specifications, processing/cutting, transportation, and current market conditions (typically ranging 30–45 TL/kg for standard S355-grade beams according to industry trends and supplier data), 40 TL/kg provides a practical midpoint for relative cost comparisons between design iterations. This figure aligns with general Turkish steel market pricing for structural grades (derived from hot-rolled coil/plate baselines around 32–38 TL/kg, plus premiums for profiled sections), and is intended as a simplified assumption for beam-link members only—excluding fabrication, welding, motors, or other components. It allows meaningful illustration of mass optimization benefits (e.g., 25% cost reduction from Iteration 1 to Iteration 3) without requiring real-time quotes, which can vary daily A concept-stage cost model is provided. The dominant cost driver for structural members is raw material mass, since all iterations use similar steel grades and similar fabrication effort.

Steel cost assumed $C = 40\,\mathrm{TL/kg}$ .

$$\begin{aligned}\text{Iteration 1:}&\ 20.80C \approx 832\,\mathrm{TL}\\ \text{Iteration 2:}&\ 19.24C \approx 769.6\,\mathrm{TL}\\ \text{Iteration 3:}&\ 15.54C \approx 621.6\,\mathrm{TL}\\ \text{Iteration 4:}&\ 6.02C \approx 240.8\,\mathrm{TL}\end{aligned}$$

Table 12: Cost comparison for Iterations 1–4 using $C = 40\,\mathrm{TL/kg}$ (beam-link members only).

| Iteration | Mass (kg) | Cost (TL) |
|---|---|---|
| 1 | 20.80 | 832.0 |
| 2 | 19.24 | 769.6 |
| 3 | 15.54 | 621.6 |
| 4 | 6.02 | 240.8 |

Iteration 4 (SHS) is the lowest-cost option for the beam-link members, but it produces a much larger deflection (see Table 13) and is therefore not selected. Iteration 2 introduces the IPEA series, which may be less common than standard IPE profiles in some supply chains. Iteration 3 remains the recommended practical design.

**Note:** This is a beam-only estimate. The full robot cost also includes motors, gearboxes, bearings, base plate machining, fasteners, welding labor, and surface finishing. However, the relative comparison between iterations remains meaningful for the structural members.

# 8 Conclusions

A 6-DOF robotic arm concept was designed in SolidWorks and verified using mechanics-of-materials hand calculations under conservative worst-case static loading: a 50 kg payload at 2 m horizontal reach.

Table 13: Iteration performance summary under the common worst-case static pose (arm horizontal, payload P=490.5 N at L=2.0 m). "Beam-link mass" includes only the equivalent straight-cantilever links.

| **Iter.** | **Beam-link mass (kg)** | **Distributed self-weight model** $w(x)$ | $V(0)$ **(N)** | $M(0)$ **(N.m)** | $\sigma_{\max}$ **(MPa)** | $\text{FoS}_{bend}$ | $\delta_{tip}$ **(mm)** |
|---|---|---|---|---|---|---|---|
| 1 | 20.80 | $w = 102.0\,\text{N/m}$ (uniform) | 694.5 | 1185.0 | 22.4 | 15.9 | 2.27 |
| 2 | 19.24 | $w = 102.0\,\text{N/m}$ for $x < 1.08\,\text{m}$; $85.3\,\text{N/m}$ for $x \geq 1.08\,\text{m}$ | 679.2 | 1161.4 | 21.9 | 16.2 | 2.27 |
| 3 | 15.54 | $w = 102.0\,\text{N/m}$ $(0 \to 0.552$ m), $79.5\,\text{N/m}$ $(0.552 \to 1.08$ m), $58.9\,\text{N/m}$ $(1.08 \to 2.0$ m) | 642.9 | 1114.2 | 23.8 | 14.9 | 3.25 |
| 4 | 6.02 | $w = 29.55\,\text{N/m}$ (uniform) | 549.6 | 1040.1 | 176.0 | 2.0 | 44.07 |

The baseline design (Iteration 1, IPE120 everywhere) is extremely conservative with $\sigma_{\max} \approx 22.38\,\text{MPa}$ and FoS $\approx 15.9$. A tapered IPE selection (Iteration 3) reduces beam-link mass to 15.54 kg (about 25% reduction) while maintaining FoS $\gg 2$ across bending, shear, axial stress, torsion, buckling, and joint hardware checks. Iteration 3 is selected as the final design because it provides the best weight/cost reduction with robust safety margin. Iteration 4 shows the minimum-cost strength-limited option (FoS$\approx$2), but produces a much larger deflection, so it is not selected.

# AI Usage Statement

AI-assisted tools were used for document structuring and formatting only. All engineering assumptions, equations, numerical calculations, and final safety conclusions were manually checked and verified by the author(s) using Mechanics of Materials course methods.

# References


[1] European Committee for Standardization (CEN), *EN 10025-2: Hot rolled products of structural steels — Part 2: Technical delivery conditions for non-alloy structural steels*, latest edition.

[2] European Committee for Standardization (CEN), *EN 10034: Structural steel I and H sections — Tolerances on shape and dimensions*, latest edition.

[3] Structolution, "IPE beam profiles (hot rolled) — Steel Section Properties," online database (IPE 80, IPE 100, IPE 120 pages used), accessed 2025-12-27. `https://structolution.com/steel-beam-properties/hot_rolled/ipe`

[4] Structolution, "IPEA 120 (hot rolled) — Steel Section Properties," online database, accessed 2025-12-27. `https://structolution.com/steel-beam-properties/hot_rolled/ipea/120`

[5] European Committee for Standardization (CEN), *EN 10210-2: Hot finished structural hollow sections of non-alloy and fine grain steels — Part 2: Tolerances, dimensions and sectional properties*, latest edition.

[6] International Organization for Standardization, *ISO 898-1: Mechanical properties of fasteners made of carbon steel and alloy steel — Part 1: Bolts, screws and studs*, latest edition.

[7] F. P. Beer, E. R. Johnston, J. T. DeWolf, and D. F. Mazurek, *Mechanics of Materials*, McGraw-Hill Education, edition used in the course.