İstanbul Bilgi Üniversitesi

Static Failure Analysis of a 6-DOF Robotic Arm

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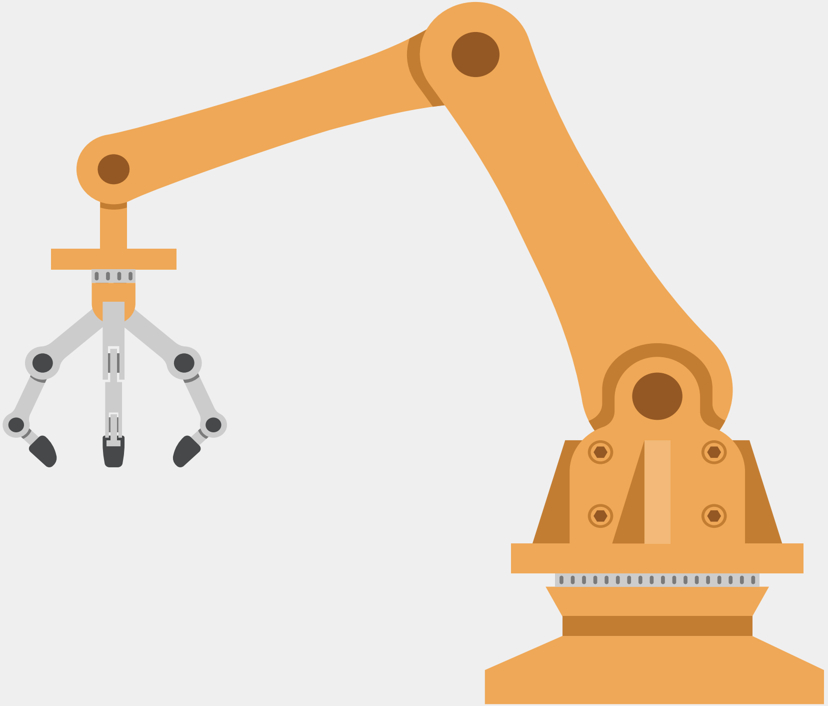

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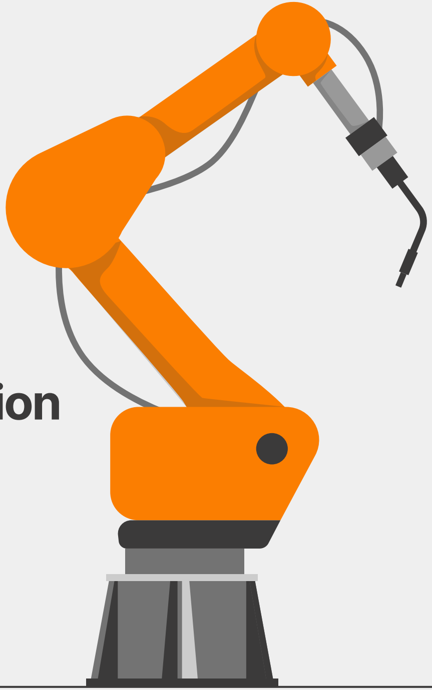

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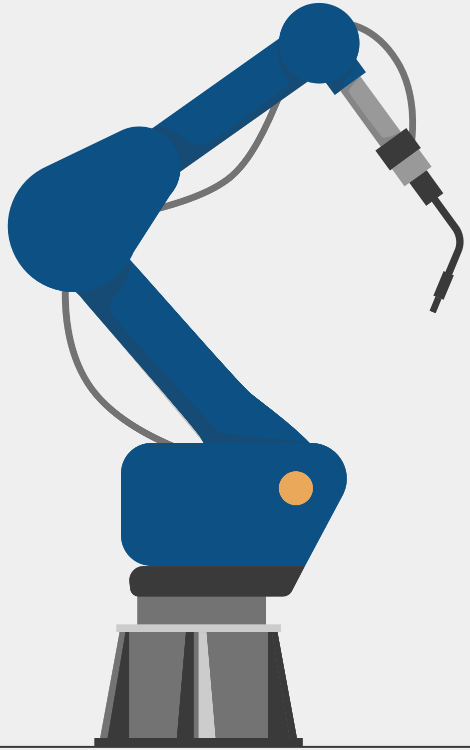
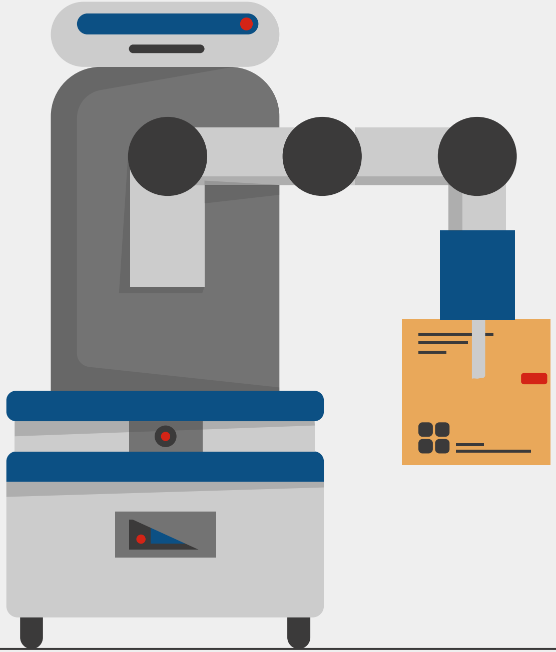




01

Design Requirements and Assumption

The Design Requirements and Assumption

Design Requirements

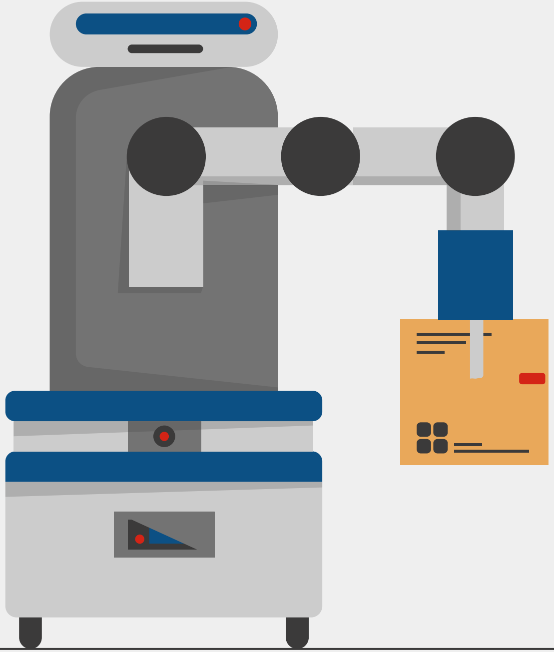
- Payload of 50 KG
- Can reach up to 2m from base axis to payload point
- At least 2 minimum Factor of Safety
- At least 2 minimum Factor of Safety
- Standard Industrial Profiles (IPE)

Structural Assumptions

- Linear elastic material behavior
- Static Loading Conditions (No dynamics, No Impacts, No Fatigue)
- Worst-case pose: horizontal reach

Load & Mass Assumptions

- Equivalent straight cantilever of length L = 2.0 m
- The weight of the beam is included as distributed load
- CAD non-beam masses excluded but bounded



02

The Robotic Arm Design

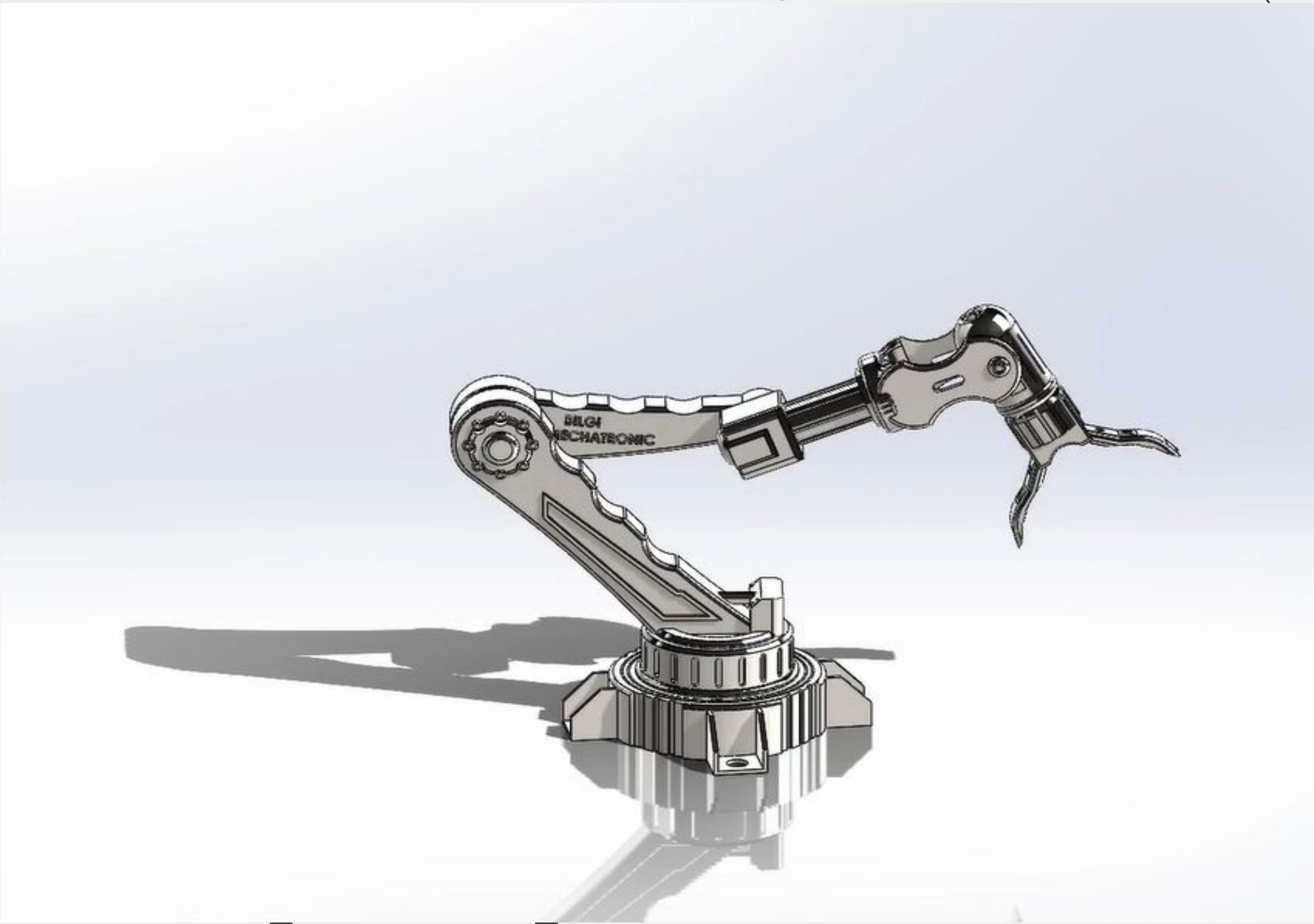


Figure 1: Overall configuration of the robotic arm (SolidWorks render).

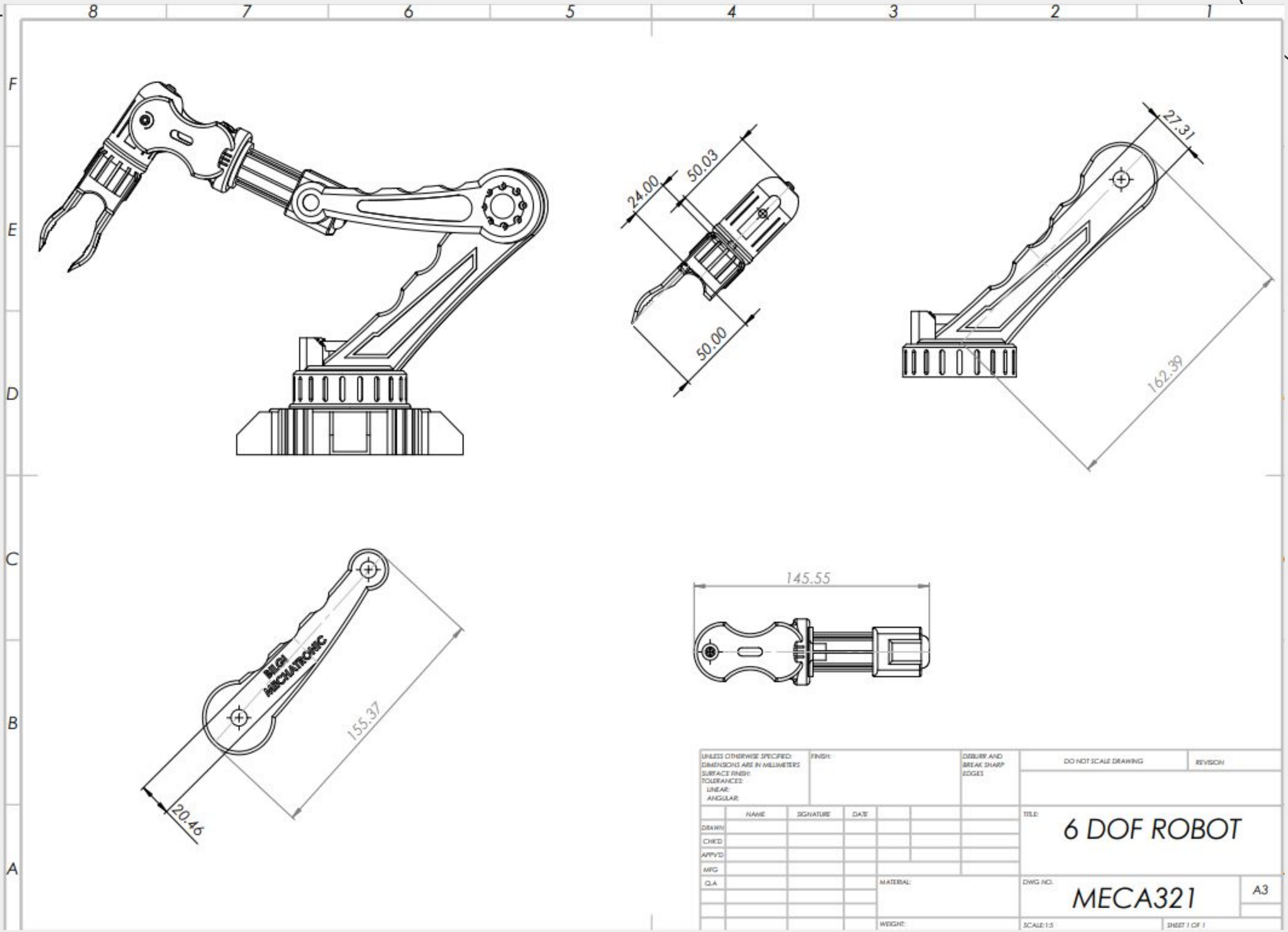
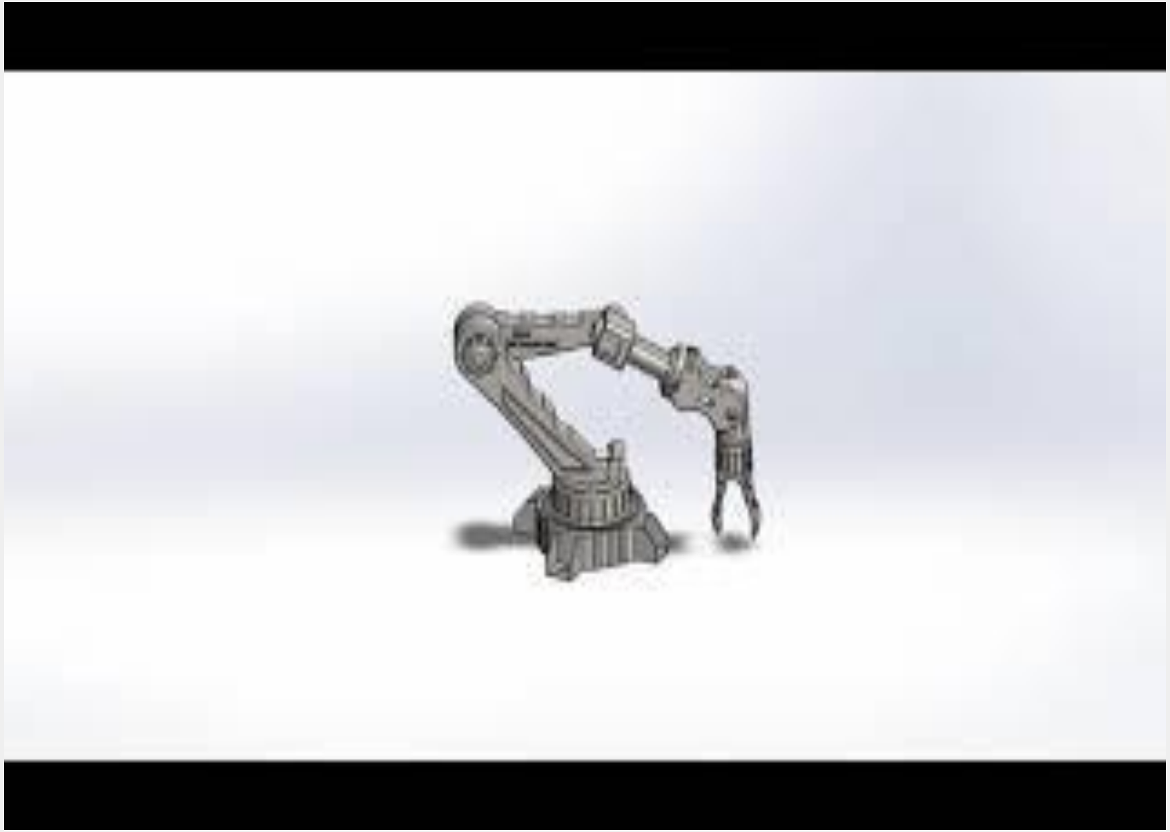
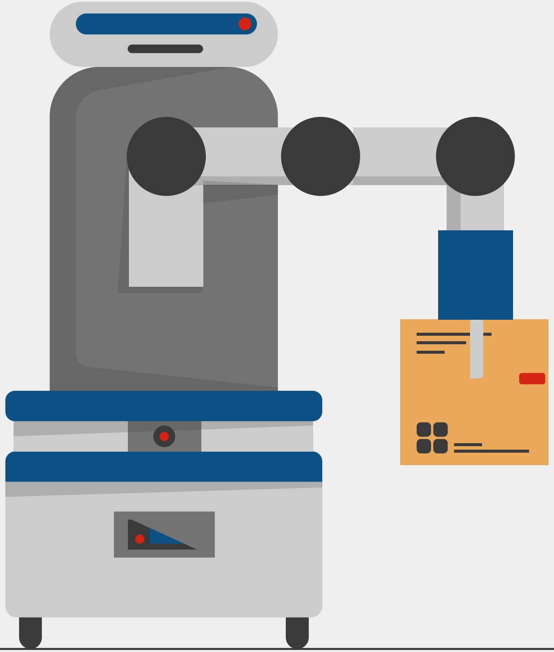


Figure 2: Technical Sketch



Video Animation



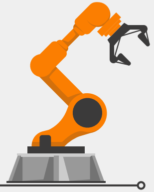
03

Material and Cross-Section Selection

Material Selection Key Properties

Component	Material	E (GPa)	σ_y (MPa)	Density (Kg/m^3)
Beam links (all iterations)	S355	210	355	7850
Pins (bounding check)	Steel (S355 Bound)	210	355	7850
Base bolts (M12, class 8.8)	ISO 8.8	210	640	7850

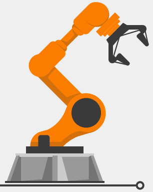
Bolt property class reference: ISO 898-1 [5].

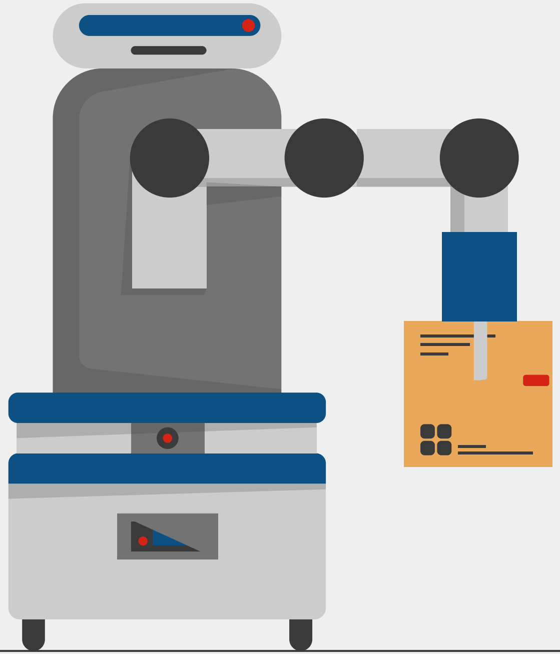


Industrial standard IPE profile data

Section	Mass (Kg / m)	W_x (cm^3)	I_x (cm^3)
IPE 120	10.4	53.0	318
IPE 100	8.1	34.2	171
IPE 80	6.0	20.0	80.1

Table 3: IPE section properties used in iteration 3





04

Structural Analysis for Static Failure

4.1 Loads and worst-case configuration

Payload (tip load):

$P = mg = 50(9.81) = 490.5N$

Worst case is the fully horizontal reach, modeled as a straight cantilever of length

$L = 2.0$ m.

Baseline self-weight (Iteration 1, all IPE 120):

$w = (10.4)(9.81) = 102N/m$

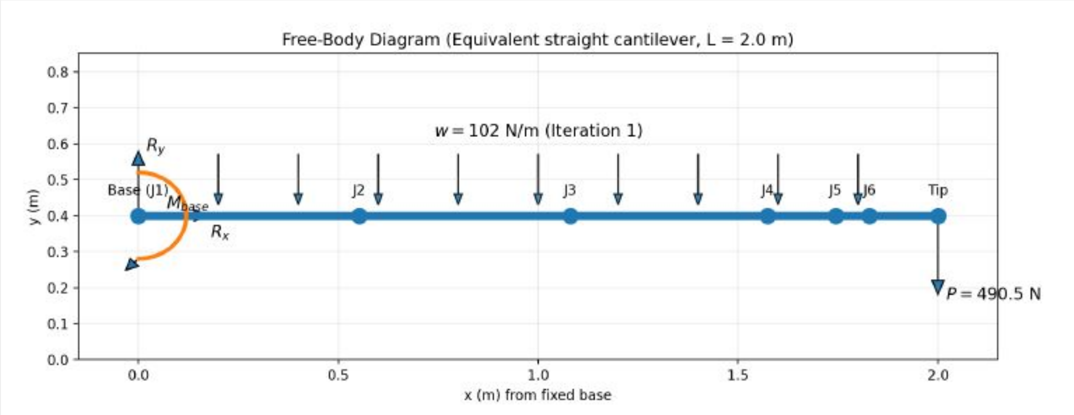


Figure 2: Free-body diagram of the equivalent straight cantilever model (L = 2.0 m) including payload P and self-weight w.

4.2 Shear force and bending moment (payload + UDL)

$V(x) = P + w(L - x),\ 0 \le x \le L$

$M(x) = P(L\text{-}x) + \frac{\omega}{2}(L - x)^2,\ 0 \le x \le L$

At the base (x = 0):

$V_{max} = P + \omega L = 490.5 + 102(2) = 694.5\ \text{N}$

$M_{max} = PL + \frac{\omega L^2}{2} = 490.5(2) + \frac{102(2^2)}{2} = 1185\ \text{N.m}$

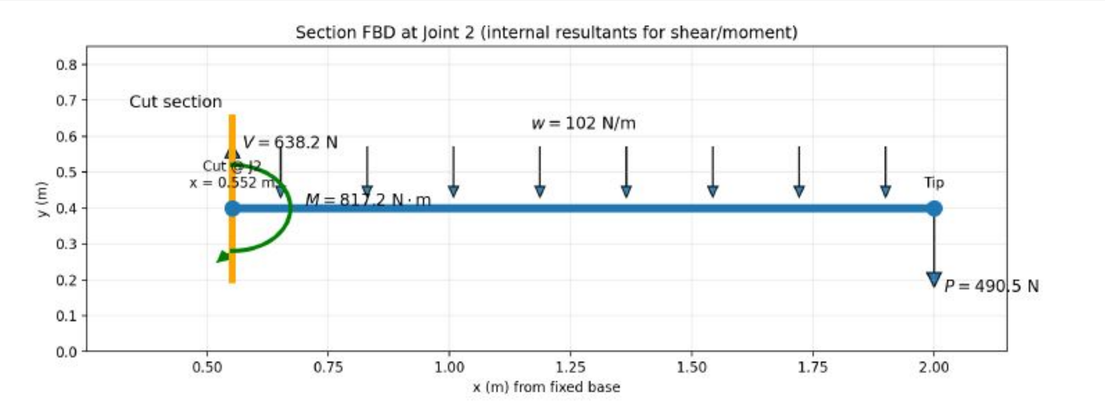


Figure 3: Section cut at Joint 2 showing internal shear V and bending moment M used for shear/moment/deflection calculations.

4.3 Bending Stress Check (Iteration 1 baseline: IPE 120 everywhere)

Convert M_{max} to mm:

M_{max} = 1185 N . m = 1.185×10^6 mm

With IPE 120, $W_x = 53\ cm^3 = 53{,}000\ mm^3$:

$$\sigma_{max} = \frac{M_{max}}{W_x} = 22.36 MPa$$

$$FoS_{bend} = \frac{\sigma_y}{\sigma_{max}} = 15.9 > 2$$

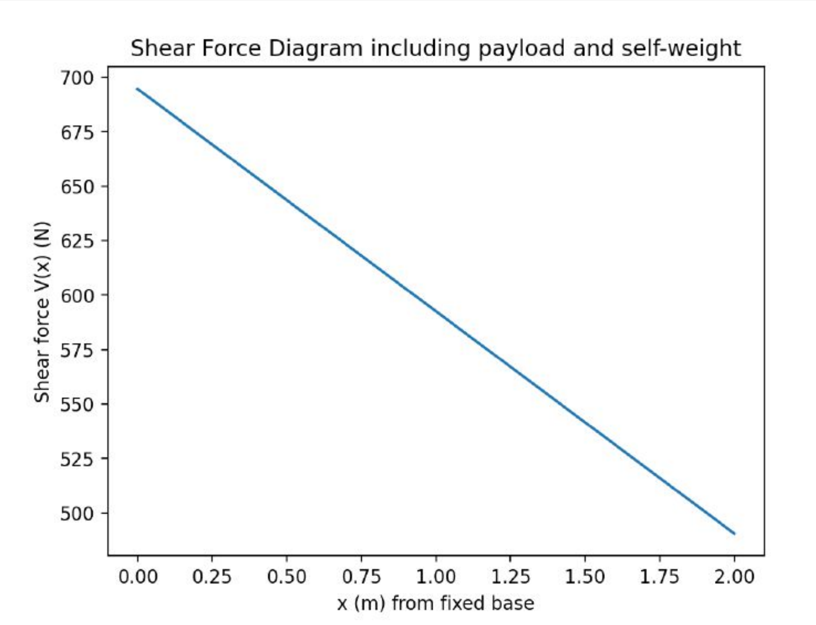


Figure 4: Shear force diagram including payload and self-weight (Iteration 1 baseline).

4.4 Deflection analysis (Iteration 1 baseline, constant EI)

IPE 120 $I_x = 318\ cm^4 = 3.18 \times 10^{-6}\ m^4$

$$\delta_p = \frac{PL^3}{3EL} = 1.96mm$$

$$\delta_w = \frac{wL^4}{8EL} = 0.31\ mm$$

$$\delta_{total} = 2.27mm$$

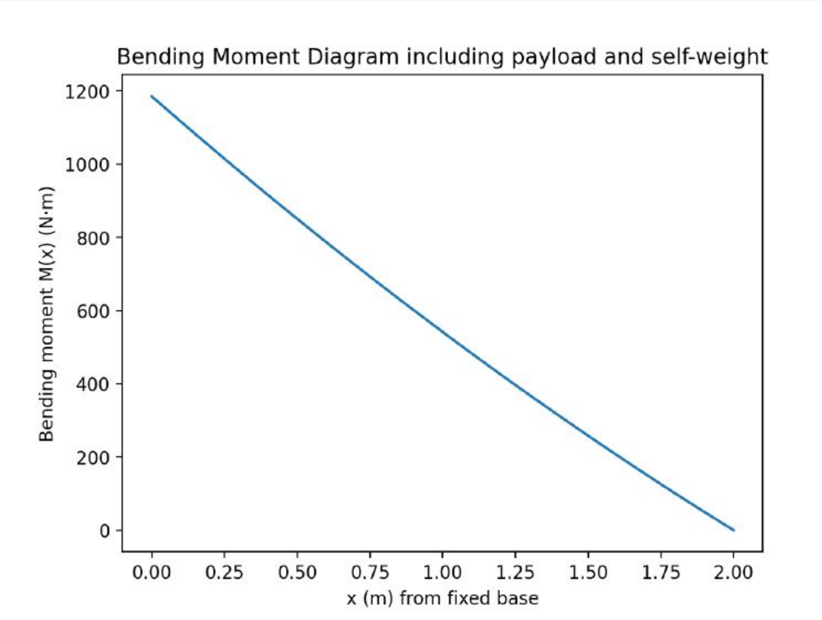


Figure 5: Bending moment diagram including payload and self-weight (Iteration 1 baseline).

4.5 Other Failure Modes

Axial stress bound

Conservative bound: $N \approx P = 490.5N$. Using IPE 120 area $A \approx 1320mm^4$:

$$\sigma_a = \frac{N}{A} = 0.372Mpa << \sigma_{allow}$$

Torsion bound (wrist shaft)

Assume eccentricity e = 0.10m:

T = Pe = 490.5 (0.10) = 49.05 N.m = 49050 N.mm

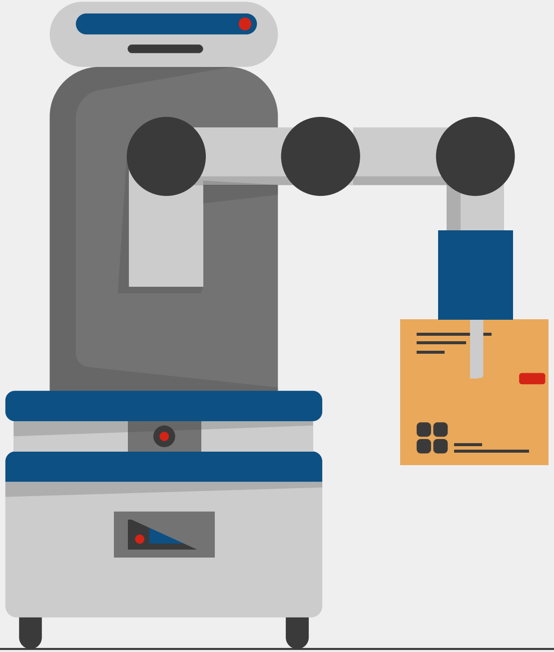
$$\tau_{max} = \frac{16T}{\pi d^3} = 61 \text{ MPa}$$

Euler buckling (bounding)

A conservative compression case is checked with N = 694.5N, Lc = 2.0m, and cantilever effective length factor K = 2. Using minor-axis inertia Imin = 27.7 cm^4 = 2.77×105 mm^4 and E = 210,000 MPa:

$$P_{cr} = \frac{\pi^2 E I_{min}}{(KL_c)^2} = 3.59\times10^4 N$$

$$Fos_{buckling} = \frac{P_{cr}}{N} = 52$$



05

Design Iteration and Optimization

Iteration Strategy and rationale

Iteration 1 (baseline)

one standard IPE profile everywhere to establish the FBDs, shear/moment/deflection, and required section modulus.

Iteration 2:

reduce mass in distal links (lower-moment region) while keeping the base link unchanged.

Iteration 3 (final):

apply a three-step taper (IPE120 → IPE100 → IPE80) to maximize mass/cost reduction while keeping deflection small.

Iteration 4 (min. cost target):

a strength-limited profile sized so that FoS ≈ 2; included to show the practical lower bound and the stiffness trade-off.

Iteration Definitions

Iteration 1

IPE120 for all links (L = 2.0m total).

Iteration 2

IPE120 for proximal links; IPEA120 for distal links (mass reduction).

Iteration 3 (Best)

IPE120 on Link 1, IPE100 on Link 2, IPE80 on Links 3–6.

Iteration 4 (Min Cost)

SHS 50×50×2 is used for all the links

Iteration 3: Updated moments

Using centroid-based summation of distributed loads for Iteration 3, the total bending moments at key stations are:

$M_{T,1} \approx 1114 N \cdot m$
$M_{T,2} \approx 774.7 N \cdot m$
$M_{T,3} \approx 476.1 N \cdot m$

Corresponding bending stresses:

$\sigma_1 = 21.0$ MPa
$\sigma_2 = 22.6$ MPa   All satisfy FoS ≥ 2.
$\sigma_3 = 23.8$ MPa

Optional: dynamic amplification and actuator sizing note

the static results are safe for gravity loads, but in real operation you multiply them by a DLF to conservatively cover dynamic effects:

$V_{dyn}(x) = DLF\ V(x)$ $M_{dyn}(x) = DLF M(x)$

5.1 Iteration 1 (IPE 120 everywhere)

Self-weight is uniform: $w = (10.4)(9.81) \approx 102 N/m$ over $L = 2m$.

J1 (Base) ($x = 0.000m$): $V = 694.5N$, $M = 1185.0N \cdot m$.

- J2 ($x = 0.552m$): $V = 638.2N$, $M = 817.2N \cdot m$.
- J3 ($x = 1.080m$): $V = 584.3N$, $M = 494.4N \cdot m$.
- J4 ($x = 1.574m$): $V = 534.0N$, $M = 218.2N \cdot m$.
- J5 ($x = 1.744m$): $V = 516.6N$, $M = 128.9N \cdot m$.
- J6 ($x = 1.829m$): $V = 507.9N$, $M = 85.4N \cdot m$.
- Tip ($x = 2.000m$): $V = 490.5N$, $M = 0.0N \cdot m$.

Station	x (m)	V (N)	M (N·m)
J1 (Base)	0.000	694.5	1185.0
J2	0.552	638.2	817.2
J3	1.080	584.3	494.4
J4	1.574	534.0	218.2
J5	1.744	516.6	128.9
J6	1.829	507.9	85.4
Tip	2.000	490.5	0.0

Table 5: Joint-by-joint internal shear and bending moment for Iteration 1 (uniform IPE 120, $w = 102N/m$, $P = 490.5N$).

5.2 Iteration 2 (IPE 120 Links 1–2, IPEA 120 Links 3–6)

Links 1–2 (up to x =1.08m) use IPE1 20: $w_{1\text{-}2}$= 102 N/m.
Links 3–6 (from x = 1.08m to the tip) use IP A120 with mass 8.7 kg/m, giving $w_{3\text{-}6}$= (8.7)(9.81) = 85.3N/m.

Station-wise results (shear and equivalent bending torque)

- J1 (Base) (x = 0.000m): V = 679.2N, M = 1161.4N · m.
- J2 (x = 0.552m): V = 622.9N, M = 802.0N · m
- J3 (x = 1.080m): V = 569.0N, M = 487.4N · m.
- J4 (x = 1.574m): V = 526.9N, M = 216.7N · m.
- J5 (x = 1.744m): V = 512.3N, M = 128.4N · m.
- J6 (x = 1.829m): V = 505.1N, M = 85.1N · m.
- Tip (x = 2.000m): V = 490.5N, M = 0.0N · m.

Station	x (m)	V (N)	M (N·m)
J1 (Base)	0.000	679.2	1161.4
J2	0.552	622.9	802.0
J3	1.080	569.0	487.4
J4	1.574	526.9	216.7
J5	1.744	512.3	128.4
J6	1.829	505.1	85.1
Tip	2.000	490.5	0.0

Table 6: Joint-by-joint internal shear and bending moment for Iteration 2 (IPE 120 on Links 1–2, IPEA 120 on Links 3–6, P = 490.5N).

5.3 Iteration 3 (IPE 120 Link 1, IPE100 Link 2, IP E80 Links 3–6)

w_1= (10.4)(9.81) = 102.0N/m, w_2= (8.1)(9.81) = 79.5N/m,
$w_{3\text{-}6}$= (6.0)(9.81) = 58.9N/m.

Station-wise results (shear and equivalent bending torque)

• J1 (Base) (x = 0.000m): V = 642.9N, M = 1114.2N · m.

• J2 (x = 0.552m): V = 586.6N, M = 774.8N · m.

• J3 (x = 1.080m): V = 544.7N, M = 476.2N · m.

• J4 (x = 1.574m): V = 515.6N, M = 214.3N · m.

• J5 (x = 1.744m): V = 505.6N, M = 127.5N · m.

• J6 (x = 1.829m): V = 500.6N, M = 84.7N · m.

• Tip (x = 2.000m): V = 490.5N, M = 0.0N · m.

Station	x (m)	V (N)	M (N·m)
J1 (Base)	0.000	642.9	1114.2
J2	0.552	586.6	774.8
J3	1.080	544.7	476.2
J4	1.574	515.6	214.3
J5	1.744	505.6	127.5
J6	1.829	500.6	84.7
Tip	2.000	490.5	0.0

Table 7: Joint-by-joint internal shear and bending moment for Iteration 3 (IPE 120 on Link 1, IPE100 on Link 2, IPE 80 on Links 3–6, P = 490.5N).

5.4 Iteration 4 (SHS 50mm×50mm×2mm everywhere).

Station-wise results (shear and equivalent bending torque)

- J1 (Base) (x = 0.000m): V = 549.6N, M = 1040.1N · m.
- J2 (x = 0.552m): V = 533.3N, M = 741.2N · m.
- J3 (x = 1.080m): V = 517.7N, M = 463.8N · m.
- J4 (x = 1.574m): V = 503.1N, M = 211.6N · m.
- J5 (x = 1.744m): V = 498.1N, M = 126.5N · m.
- J6 (x = 1.829m): V = 495.6N, M = 84.3N · m.
- Tip (x = 2.000m): V = 490.5N, M = 0.0N · m.

Station	x (m)	V (N)	M (N·m)
J1 (Base)	0.000	549.6	1040.1
J2	0.552	533.3	741.2
J3	1.080	517.7	463.8
J4	1.574	503.1	211.6
J5	1.744	498.1	126.5
J6	1.829	495.6	84.3
Tip	2.000	490.5	0.0

Table 8: Joint-by-joint internal shear and bending moment for Iteration 4 (SHS 50mm×50mm×2 mm, P = 490.5N).

5.3.1 Bounding joint pin shear/bearing check (concept-stage)[12]

Using the concept-stage joint pin diameter d_p = 16mm and lug thickness t = 10mm (double shear, two lugs), the pin shear stress and lug bearing pressure are:

$$\tau_p = \frac{V}{2A_p}, \qquad A_p = \frac{\pi d_p^2}{4}, \qquad p_b = \frac{V}{2td_p}.$$

These stresses are far below the S355 yield strength; therefore, under the static-gravity
load case, joint pin shear/bearing is non-governing. Dynamic loads, impact, and fatigue
should be considered in a detailed design phase.

Joint	V (N)	τ_p (MPa)	p_b (MPa)
Joint 2	586.6	1.459	1.833
Joint 3	544.7	1.354	1.702
Joint 4	515.6	1.282	1.611
Joint 5	505.6	1.257	1.580
Joint 6	500.6	1.245	1.564

Table 9: Pin shear stress and lug bearing pressure at the main joints (final design: Iteration 3 loads from Table 7).

5.3-2 Iteration 3 deflection (piecewise EI)

A piecewise stiffness estimate is performed using the unit-load method:

$$\delta(L) = \int_0^L \frac{M(x)\, m(x)}{EI(x)}\, dx$$

where m(x) = L–x is the unit-load bending moment at the tip. With piecewise I(x) for

IPE 120/IPE 100/IPE 80 segments and including payload plus self-weight, the predicted

tip deflection is:

$\delta_3 \approx 3.25$mm

This remains small relative to the 2m reach and does not govern static failure.

5.4 Iteration 4: Lowest-cost strength-limited design (target FoS ≈2)

Iteration 4 explores the lowest-cost option that still satisfies the project requirement FoS ≥ 2. Instead of selecting an IPE section that produces a large safety margin, a standard square hollow section (SHS) is sized so that the base bending stress approaches σallow = σy/2. Structural hollow sections are standardized in EN 10210 [5]. A practical standard size is SHS 50mm×50mm×2mm with:

$A=384mm^2$, $I=147{,}712mm^4$, $W=5908mm^3$

Property	Formula (square tube)	Value
Outer width	b	50
Wall thickness	t	2
Area	$A = b^2 - (b-2t)^2$	384
Second moment	$I_x = \frac{b^4 - (b-2t)^4}{12}$	147,712
Section modulus	$W_x = \frac{I_x}{b/2}$	5908

Table 10: SHS 50mm×50mm×2mm section-property calculation (used in Iteration 4).

5.4 Iteration 4: Lowest-cost strength-limited design (target FoS ≈2)

Using the same cantilever model with L = 2m, payload P = 490.5N, and self-weight w =m′g:

$$M_4(0) = PL + \frac{wL^2}{2} = 1040.1Nm, \qquad \sigma_{4,\max} = \frac{M_4(0)}{W} \approx 176.04MPa, \qquad n_4 \approx 2.02.$$

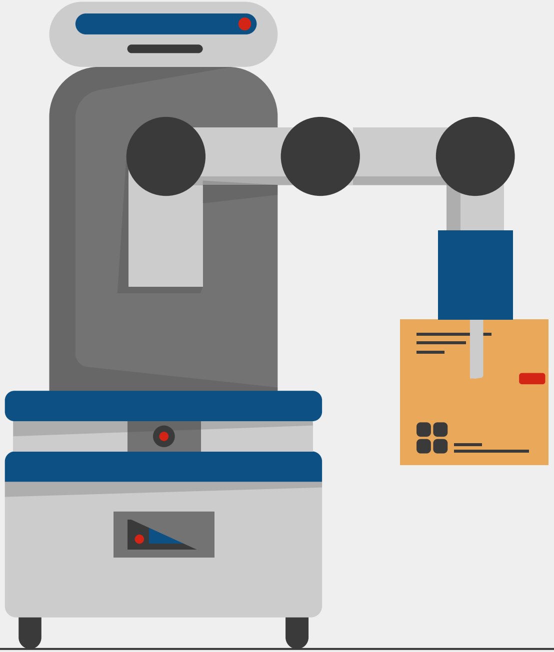
This meets the minimum safety requirement. However, the stiffness penalty is significant:

$$\delta_4 = \frac{PL^3}{3EI} + \frac{wL^4}{8EI} \approx 44.1mm.$$

Therefore, Iteration 4 is the *lowest-cost strength-compliant* case, but it is not selected as the final design due to its much larger deflection (precision/stiffness concerns).

Design	Mass (kg)	Max Stress (MPa)	FOS
Iteration 1: IPE120 everywhere	20.80	22.38	15.9
Iteration 2: IPE120 + IPEA120	19.24	21.90	16.2
Iteration 3 (recommended): IPE120 + IPE100 + IPE80	15.54	23.80	14.9
Iteration 4 (min. cost): SHS 50×50×2	6.02	176.04	2.02

Table 11: Design iteration and optimization summary.



06

Cost Analysis

6.1 Cost Analysis

We used **40 TL/kg** as a conservative midpoint for S355 steel in Türkiye (≈30–45 TL/kg, coil/plate ≈32–38 + section premiums). Applied only to beam-link mass, excluding fabrication/components. Shows optimization (e.g., 25% cost cut Iteration 1→4). Raw mass is the dominant cost driver.

Iteration	Mass (kg)	Cost (TL)
1	20.80	832.0
2	19.24	769.6
3	15.54	621.6
4	6.02	240.8

Table 12: Cost comparison for Iterations 1–4 using C = 40 TL/kg (beam-link members only).

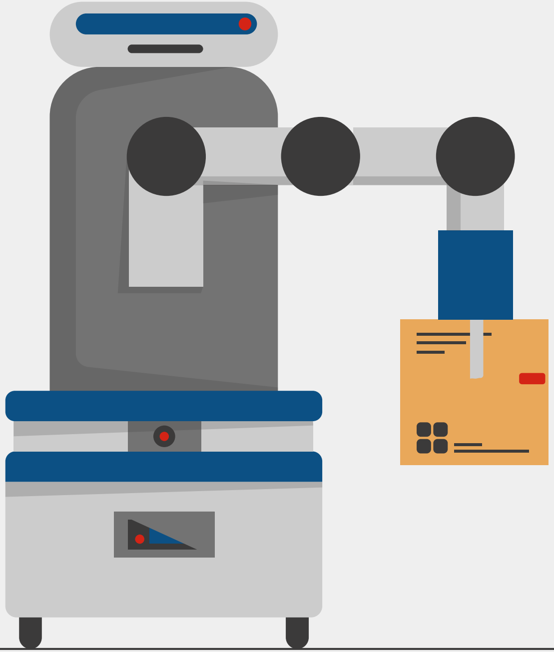
Steel cost assumed C = 40TL/kg

Iteration 1: 20.80C ≈ 832TL
Iteration 2: 19.24C ≈ 769.6TL
Iteration 3: 15.54C ≈ 621.6TL
Iteration 4: 6.02C ≈ 240.8TL

6.2 Cost Analysis Conclusions

Iteration 4 (SHS) is the lowest-cost option for the beam-link members, but it produces a much larger deflection (see Table 13) and is therefore not selected. Iteration 2 introduces the IPEA series, which may be less common than standard IPE profiles in some supply chains. Iteration 3 remains the recommended practical design.

Note: This is a beam-only estimate. The full robot cost also includes motors, gear boxes, bearings, base plate machining, fasteners, welding labor, and surface finishing. However, the relative comparison between iterations remains meaningful for the structural members.



07

Conclusion

Conclusion

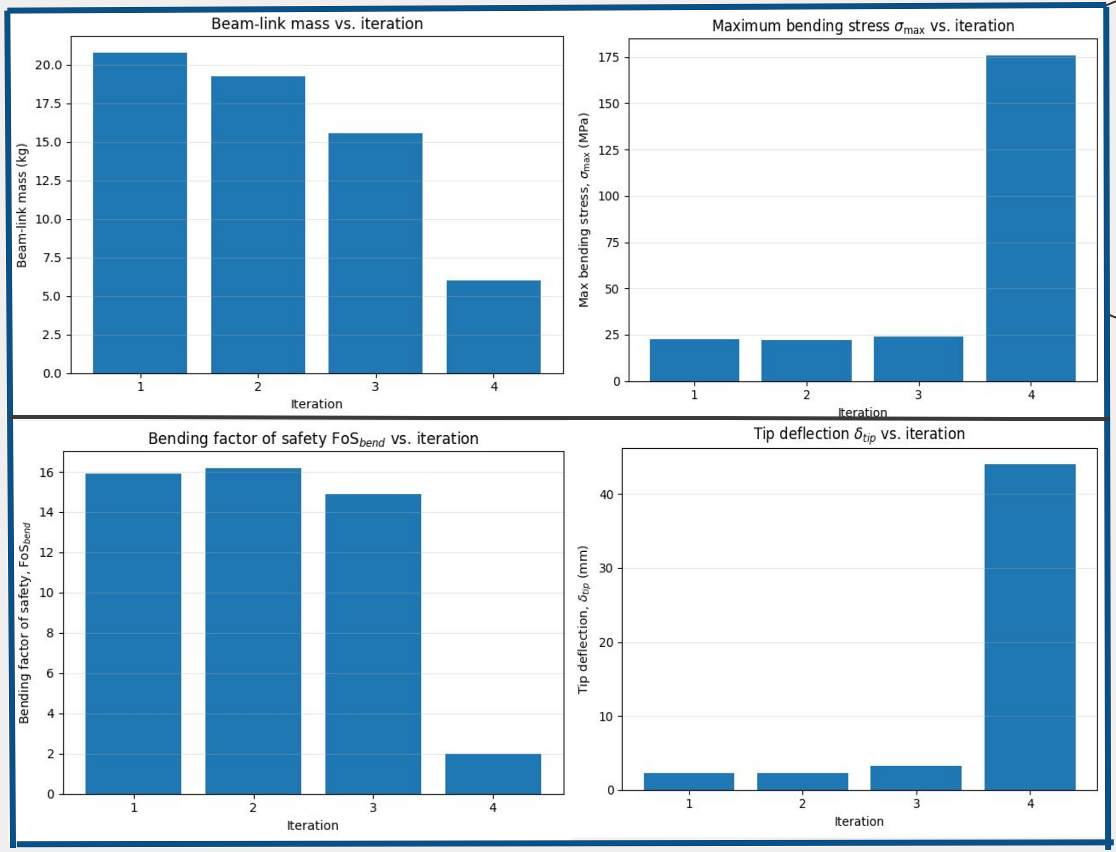
A 6-DOF robotic arm concept was designed in SolidWorks and verified using mechanics-of-materials hand calculations under conservative worst-case static loading: a 50kg pay-load at 2m horizontal reach.

Iter.	Beam-link mass (kg)	Distributed self-weight model $w(x)$	$V(0)$ (N)	$M(0)$ (N.m)	$\sigma_{\max}$ (MPa)	FoS_{bend}	δ_{tip} (mm)
1	20.80	$w = 102.0\,\text{N/m}$ (uniform)	694.5	1185.0	22.4	15.9	2.27
2	19.24	$w = 102.0\,\text{N/m}$ for $x < 1.08\,\text{m}$; $85.3\,\text{N/m}$ for $x \geq 1.08\,\text{m}$	679.2	1161.4	21.9	16.2	2.27
3	15.54	$w = 102.0\,\text{N/m}$ $(0 \to 0.552$ m), $79.5\,\text{N/m}$ $(0.552 \to 1.08$ m), $58.9\,\text{N/m}$ $(1.08 \to 2.0$ m)	642.9	1114.2	23.8	14.9	3.25
4	6.02	$w = 29.55\,\text{N/m}$ (uniform)	549.6	1040.1	176.0	2.0	44.07

Table 13: Iteration performance summary under the common worst-case static pose (arm horizontal, payload P=490.5N at L=2.0m). “Beam-link mass” includes only the equivalent straight-cantilever links

Conclusion

The baseline configuration (Iteration 1, IPE 120 throughout) is highly conservative, exhibiting a maximum bending stress σmax of approximately 22.4 MPa and a bending factor of safety of about 15.9. The tapered IPE configuration (Iteration 3) achieves a beam-link mass reduction to 15.54 kg, corresponding to an approximate 25% decrease, while maintaining a factor of safety well above 2 for all governing failure modes. Consequently, Iteration 3 is selected as the final design, as it offers the most effective balance between structural efficiency, cost, and safety. Iteration 4 represents a strength-limited minimum-mass option but is not selected due to its excessive deflection.



Resources

- [1] European Committee for Standardization (CEN), EN 10025-2: Hot rolled products of structural steels — Part 2: Technical delivery conditions for non-alloy structural steels, latest edition.
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Thanks!

Any questions?