Thermal Management of Multi-Core Processor: Cooling Analysis with Gyroid and SchwarzD Structure Heat Sink

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1 Introduction

This report presents a complete thermal analysis of multi-core processor cooling systems using advanced heat sink designs. We focus on Triply Periodic Minimal Surface (TPMS) structures, specifically Gyroid and Schwarz-D designs, for efficient heat removal.

Modern processors generate high heat loads that must be removed to prevent thermal failure. The maximum junction temperature is typically limited to 85°C. Our study compares different TPMS heat sink designs for both single and dual CPU configurations.

1.1 Problem Statement

We need to design a cooling system for:

- Two processors: CPU1 (20W) and CPU2 (15W)
- Each processor area: 1 cm × 1 cm
- Maximum junction temperature: 85°C
- Ambient temperature: 35°C

1.2 Objectives

1. Calculate thermal resistance networks for different configurations
2. Design and compare TPMS heat sink structures
3. Run thermal simulations with and without forced cooling
4. Analyze costs for different designs
5. Find the best cooling solution

2 Materials and Methods

2.1 System Parameters

Table 1 shows the main system parameters used in this study.

Table 1: System Parameters

Parameter	**Value**
Processor 1 Power (P1)	20 W
Processor 2 Power (P2)	15 W
Processor Area	1 × 1 cm^2
Maximum Junction Temperature	85°C
Ambient Temperature	35°C
TIM Thickness	100 μm
TIM Thermal Conductivity	10 W/m·K
Copper Conductivity	400 W/m·K
Aluminum Conductivity	177 W/m·K
Convection Coefficient Range	7 - 20,000 W/m^2·K

2.2 Material Properties

Different materials are used in the cooling system. Table 2 lists their thermal properties.

Table 2: Material Properties

Material	k [W/m·K]	ρ [g/cm³]	cp [J/g·°C]	Application
Silicon (Si)	150	2.328	0.678	CPU Die
TIM	56	7.850	0.480	Interface
Copper	400	8.940	0.385	Heat Spreader
Aluminum	177	2.700	0.896	Heat Sink

3 Thermal Resistance Network Analysis

3.1 Single CPU Configuration

For a single active CPU, heat flows through a series of thermal resistances from the junction to ambient air.

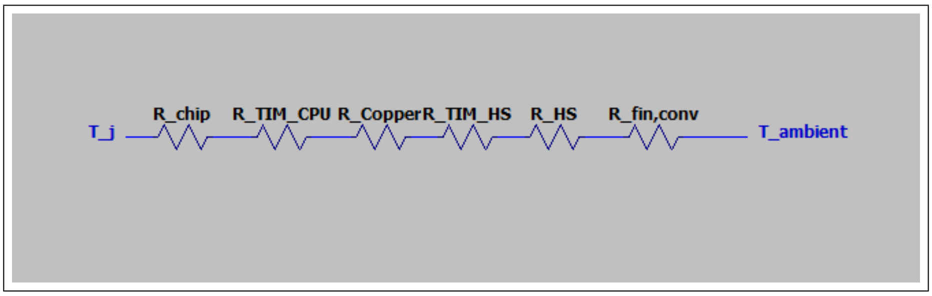


Figure 1: Single CPU Thermal Resistance Network

The total resistance must be less than the maximum allowable value:

$$R_{max} = \frac{\Delta T}{Q} = \frac{85 - 35}{20} = 2.5 \text{ °C/W} \tag{1}$$

3.1.1 Resistance Calculations

Each component contributes to the total thermal resistance:

Chip Resistance:

$$R_{chip} = \frac{L_{chip}}{k_{chip} \cdot A_{chip}} = \frac{0.5 \times 10^{-3}}{150 \times 10^{-4}} = 0.0333 \text{ °C/W} \tag{2}$$

TIM Resistance (CPU side):

$$R_{TIM,CPU} = \frac{L_{TIM}}{k_{TIM} \cdot A_{TIM}} = \frac{10^{-4}}{10 \times 10^{-4}} = 0.1 \text{ °C/W} \tag{3}$$

Copper Heat Spreader:

$$R_{copper} = \frac{L_{copper}}{k_{copper} \cdot A_{copper}} = \frac{3 \times 10^{-3}}{400 \times (25 \times 36 \times 10^{-6})} = 0.00833 \text{ °C/W} \tag{4}$$

TIM Resistance (Heat Sink side):

$$R_{TIM,HS} = \frac{L_{TIM}}{k_{TIM} \cdot A_{TIM,HS}} = 0.011\ ^\circ\text{C/W} \tag{5}$$

Heat Sink Base:

$$R_{HS} = \frac{L_{HS}}{k_{HS} \cdot A_{HS}} = 0.044\ ^\circ\text{C/W} \tag{6}$$

Convection Resistance:

$$R_{fin,conv} = \frac{1}{h_{conv} \cdot A_{conv}} = \frac{1}{h \cdot A_{conv}} \tag{7}$$

3.1.2 Gyroid Heat Sink - Single CPU

For the Gyroid design with surface area 15,452 mm²:

$$R_{fin,conv} = \frac{62.5}{h}\ ^\circ\text{C/W} \tag{8}$$

$$R_{total} = 0.197 + \frac{62.5}{h} \tag{9}$$

To meet the temperature limit:

$$2.5 = 0.197 + \frac{62.5}{h} \Rightarrow h = 27.139\ \text{W/m}^2\text{K} \tag{10}$$

3.1.3 Schwarz-D Heat Sink - Single CPU

For the Schwarz-D design with surface area 19,072 mm²:

$$R_{fin,conv} = \frac{52.63}{h}\ ^\circ\text{C/W} \tag{11}$$

$$R_{total} = 0.197 + \frac{52.63}{h} \tag{12}$$

Required convection coefficient:

$$h = 22.853\ \text{W/m}^2\text{K} \tag{13}$$

3.2 Two CPU Configuration

When both CPUs are active, they share the heat sink and convection path.

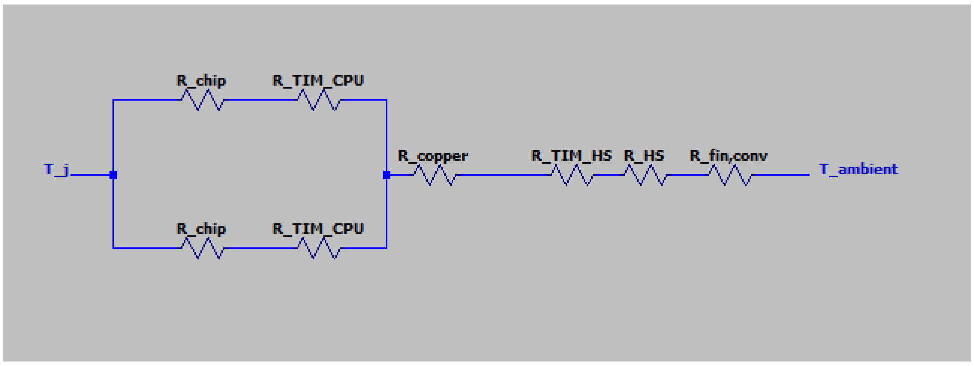


Figure 2: Two CPU Thermal Resistance Network - Two parallel branches (CPU1 and CPU2) merge at common heat sink base, then share heat sink and convection resistance to ambient

The thermal paths are:
CPU1 Path:

$$R_{path,1} = R_{TIM,1} + R_{Cu,1} + R_{pipe,1} \tag{14}$$

CPU2 Path:

$$R_{path,2} = R_{TIM,2} + R_{Cu,2} + R_{pipe,2} \tag{15}$$

Common Path:

$$R_{common} = R_{HS} + \frac{R_{conv} \cdot R_{rad}}{R_{conv} + R_{rad}} \tag{16}$$

Junction temperatures are:

$$T_{j,1} = T_\infty + P_1 \cdot R_{path,1} + (P_1 + P_2) \cdot R_{common} \tag{17}$$

$$T_{j,2} = T_\infty + P_2 \cdot R_{path,2} + (P_1 + P_2) \cdot R_{common} \tag{18}$$

3.2.1 Schwarz-D - Two CPU

Maximum allowable resistance for 35W total:

$$R_{max} = \frac{85 - 35}{35} = 1.429\ ^\circ\text{C/W} \tag{19}$$

Parallel chip resistances:

$$\frac{1}{R_{eq}} = \frac{1}{0.133} + \frac{1}{0.133} \Rightarrow R_{eq} = 0.0665\ ^\circ\text{C/W} \tag{20}$$

Total resistance:

$$R_{total} = 0.0926 + \frac{31.25}{h} \tag{21}$$

Required h:

$$h = 23.384\ \text{W/m}^2\text{K} \tag{22}$$

3.2.2 Gyroid - Two CPU

For Gyroid with larger surface area (29,229 mm²):

$$R_{total} = 0.0926 + \frac{34.48}{h} \tag{23}$$

Required h:

$$h = 25.803 \text{ W/m}^2\text{K} \tag{24}$$

Note: The detailed step-by-step thermal resistance calculations using nodal analysis methodology for the two CPU configuration are presented in Appendix D, which provides the correct physical approach for heat flow analysis when multiple heat sources merge at a common junction.

4 TPMS Heat Sink Design

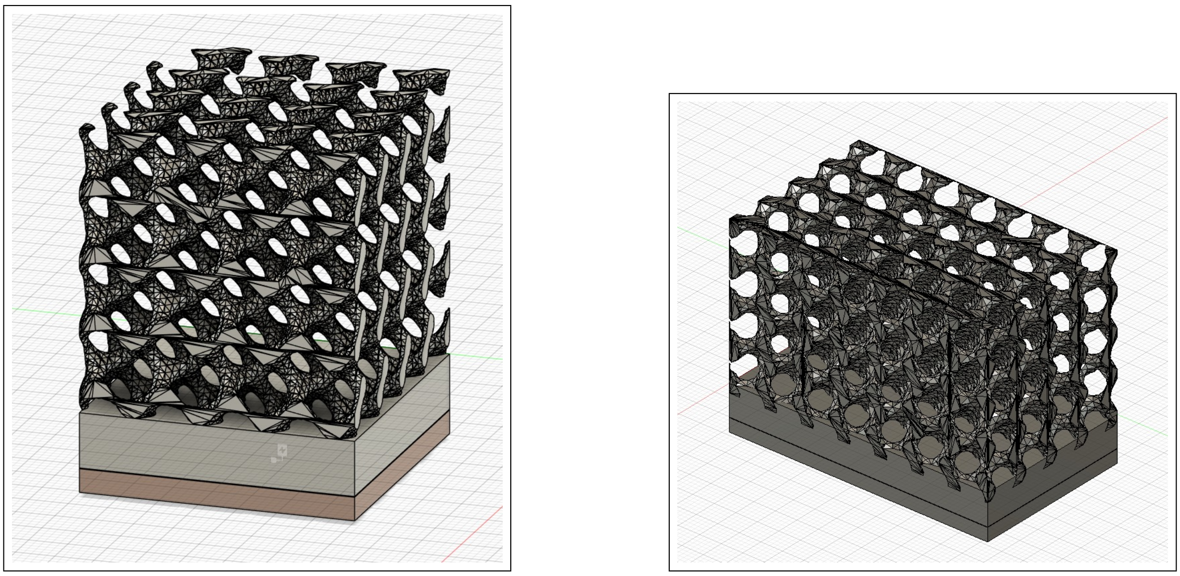
4.1 What are TPMS Structures?

Triply Periodic Minimal Surfaces (TPMS) are special geometric shapes with:

- Zero mean curvature at every point
- High surface area to volume ratio
- Continuous air flow channels
- Can be made by 3D printing

4.1.1 Gyroid Structure

The Gyroid is defined by the equation:



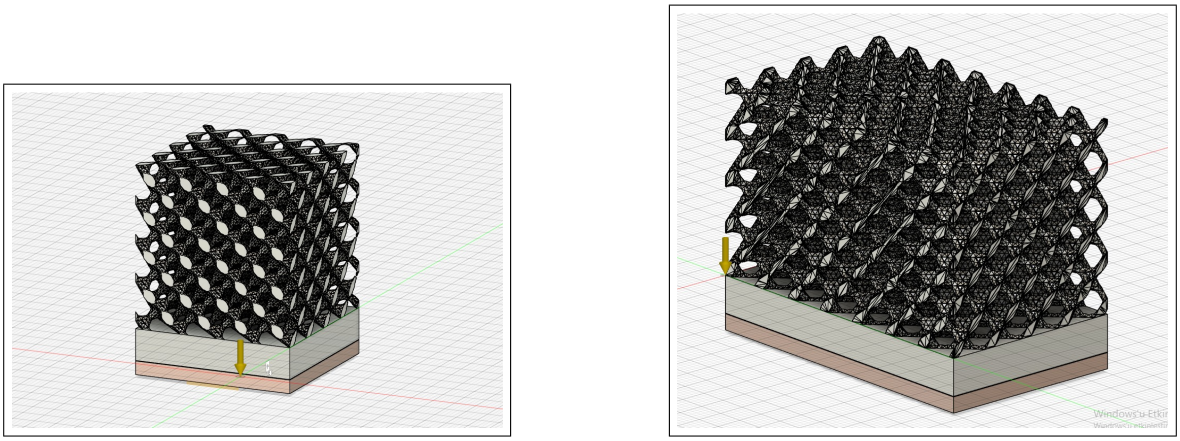
(a) Single CPU configuration

(b) Dual CPU configuration

Figure 3: Gyroid Heat Sink Structure

4.1.2 Schwarz-D (Diamond) Structure

The Schwarz-D is defined by:



(a) Single CPU

(b) Dual CPU

Figure 4: Schwarz-D Heat Sink Structure

4.2 Design Parameters

Four different heat sink designs were created:

Table 3: TPMS Heat Sink Design Parameters

Design	Surface Area [mm^2]	Volume [mm^3]	Mass [kg]	A/V Ratio [1/mm]
Gyroid 2 CPU	29,228.81	19,698.07	0.155	1.484
Gyroid 1 CPU	15,452.44	9,057.74	0.071	1.706
Schwarz-D 2 CPU	32,299.87	15,460.53	0.121	2.089
Schwarz-D 1 CPU	19,071.72	9,034.60	0.071	2.111

Key finding: Schwarz-D structures have higher surface area to volume ratios, which means better heat transfer.

5 Thermal Analysis Results

All thermal simulations were done in Fusion 360 with:

- Heat sources: CPU1 = 20W, CPU2 = 15W
- Ambient temperature: 35°C
- Material: Aluminum AlSi10Mg
- Two cases: with fan and without fan

5.1 Two CPU Configuration Results

5.1.1 Gyroid - With Fan

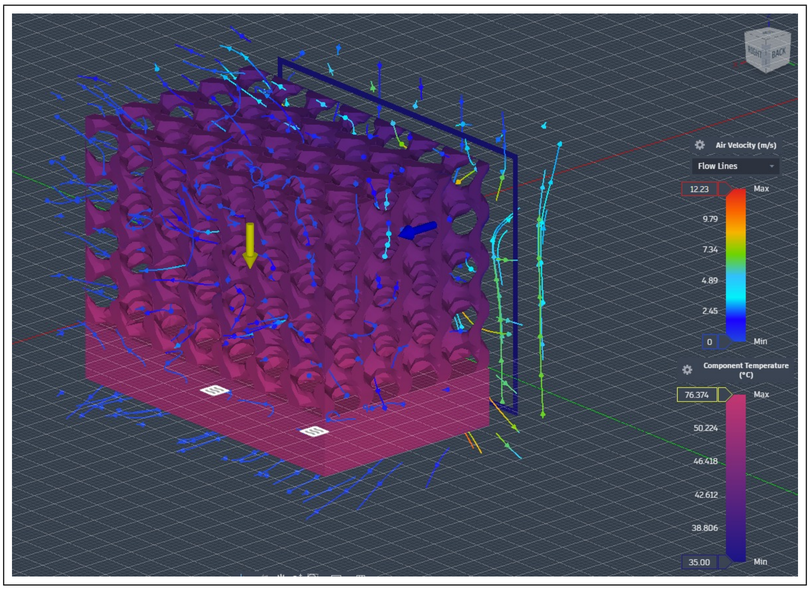


Figure 5: Gyroid 2 CPU Thermal Analysis with Fan

Results:

- Maximum temperature: 76.37°C
- Minimum temperature: 35.00°C
- Temperature rise: 41.37°C
- Maximum air velocity: 12.23 m/s
- Status: **PASS** (below 85°C limit)

5.1.2 Gyroid - No Fan

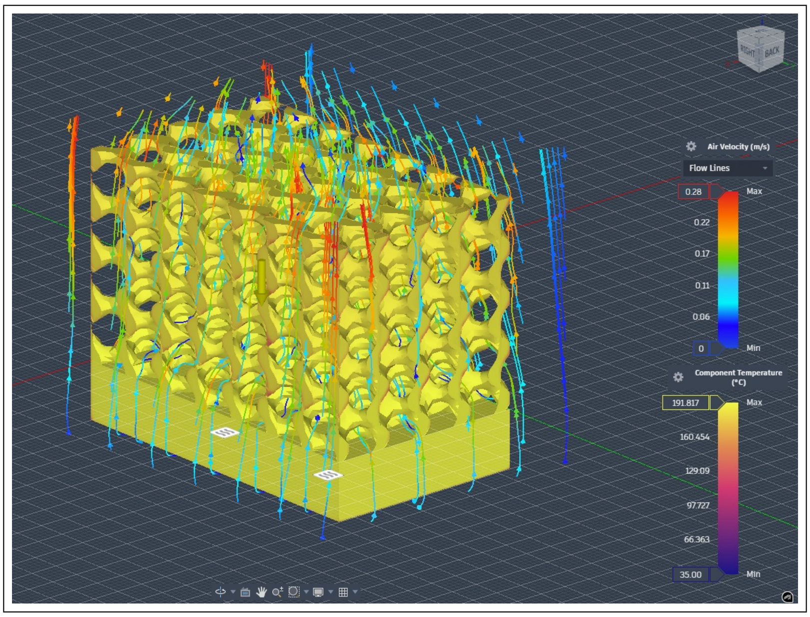


Figure 6: Gyroid 2 CPU Thermal Analysis without Fan

Results:

- Maximum temperature: 191.82°C
- Minimum temperature: 35.00°C
- Temperature rise: 156.82°C
- Maximum air velocity: 0.28 m/s (natural convection)
- Status: **FAIL** (exceeds 85°C limit)

5.1.3 Schwarz-D - With Fan

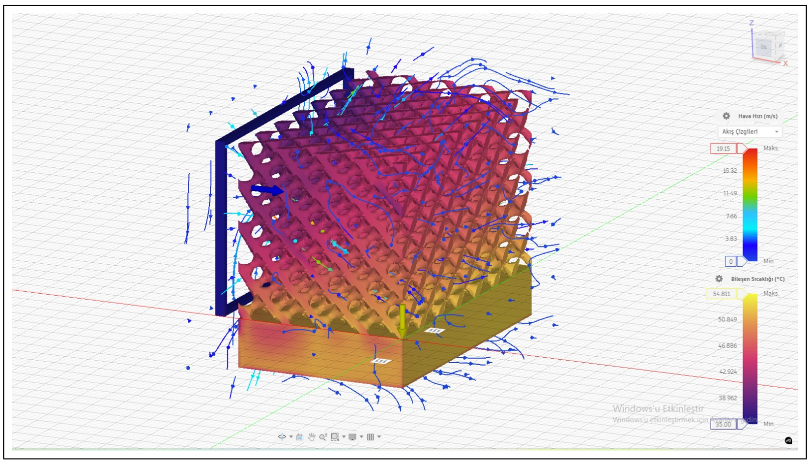


Figure 7: Schwarz-D 2 CPU Thermal Analysis with Fan

Results:

- Maximum temperature: 54.81°C
- Minimum temperature: 35.00°C
- Temperature rise: 19.81°C
- Maximum air velocity: 19.15 m/s
- Status: **PASS** (well below 85°C limit)

5.1.4 Schwarz-D - No Fan

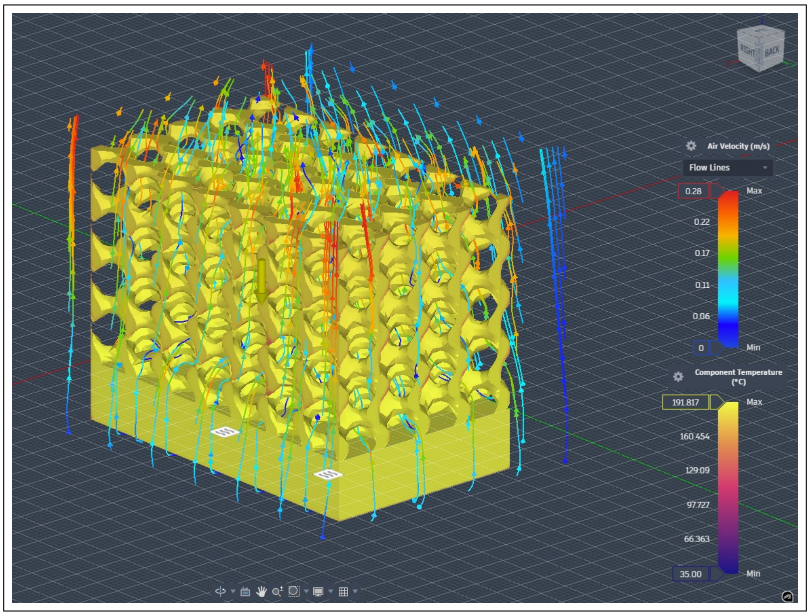


Figure 8: Schwarz-D 2 CPU Thermal Analysis without Fan

Results:

- Maximum temperature: 199.00°C
- Minimum temperature: 35.00°C
- Temperature rise: 164.00°C
- Maximum air velocity: 0.26 m/s (natural convection)
- Status: **FAIL** (exceeds 85°C limit)

5.2 Single CPU Configuration Results

5.2.1 Gyroid - With Fan

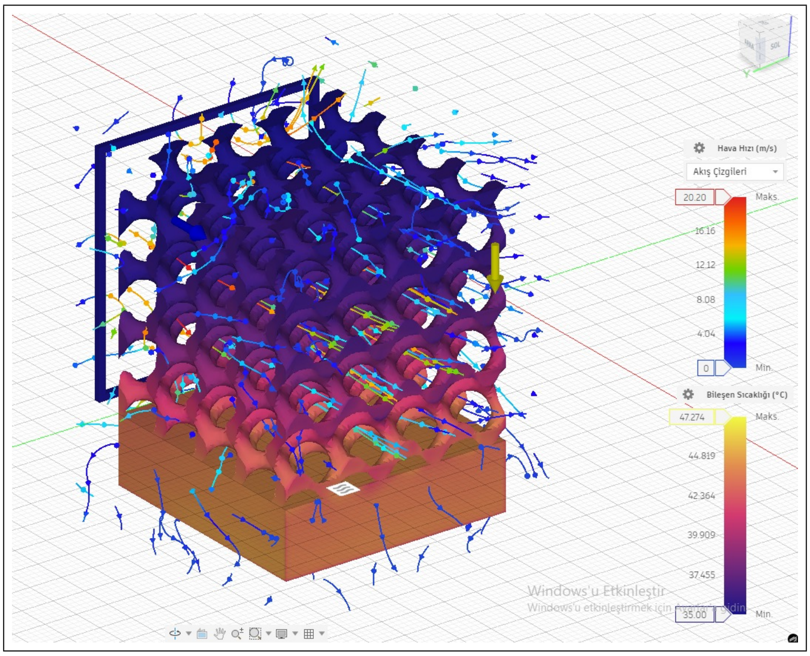


Figure 9: Gyroid 1 CPU Thermal Analysis with Fan

Results:

- Maximum temperature: 47.27°C
- Temperature rise: 12.27°C
- Maximum air velocity: 20.20 m/s
- Status: **PASS**

5.2.2 Gyroid - No Fan

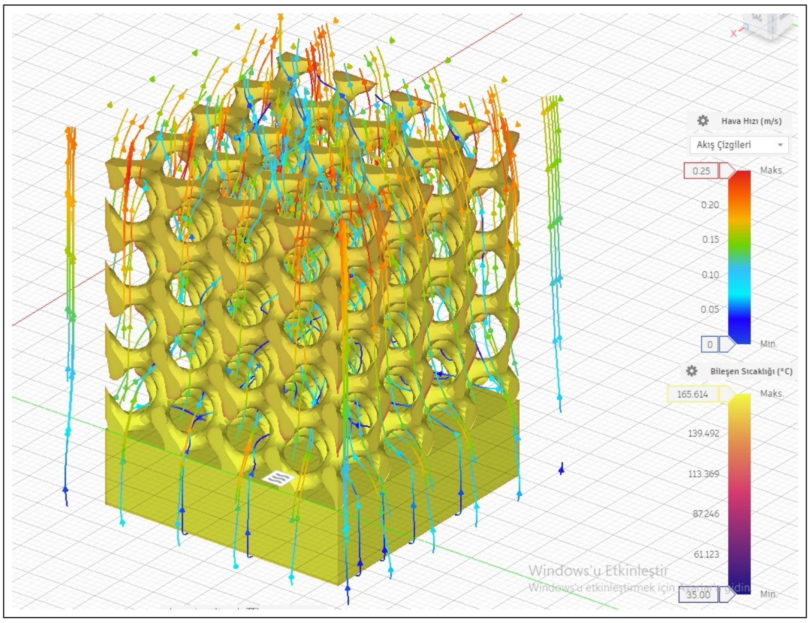


Figure 10: Gyroid 1 CPU Thermal Analysis without Fan

Results:

- Maximum temperature: 165.61°C
- Temperature rise: 130.61°C
- Maximum air velocity: 0.25 m/s
- Status: **FAIL**

5.2.3 Schwarz-D - With Fan

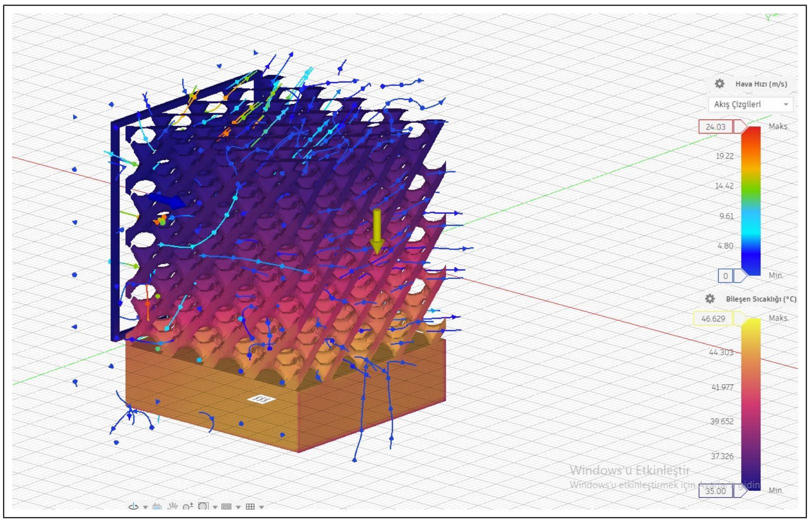


Figure 11: Schwarz-D 1 CPU Thermal Analysis with Fan

Results:

- Maximum temperature: 46.63°C
- Temperature rise: 11.63°C
- Maximum air velocity: 24.03 m/s
- Status: **PASS**

5.2.4 Schwarz-D - No Fan

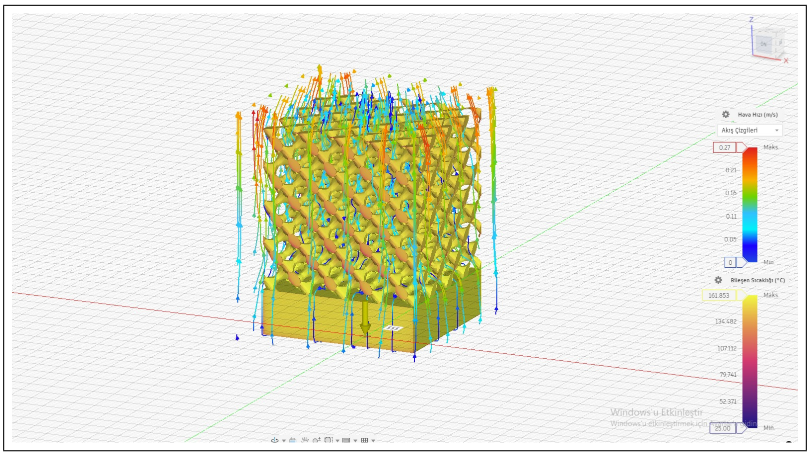


Figure 12: Schwarz-D 1 CPU Thermal Analysis without Fan

Results:

- Maximum temperature: 161.85°C
- Temperature rise: 136.85°C
- Maximum air velocity: 0.27 m/s
- Status: **FAIL**

5.3 Complete Results Comparison

Table 4: Thermal Analysis Results Summary

Design	Cooling	T_max [°C]	T_min [°C]	ΔT [°C]	Status
2 CPU Configuration (35W)					
Gyroid	Fan	76.37	35.00	41.37	PASS
Gyroid	No Fan	191.82	35.00	156.82	FAIL
Schwarz-D	Fan	54.81	35.00	19.81	PASS
Schwarz-D	No Fan	199.00	35.00	164.00	FAIL
1 CPU Configuration (20W)					
Gyroid	Fan	47.27	35.00	12.27	PASS
Gyroid	No Fan	165.61	35.00	130.61	FAIL
Schwarz-D	Fan	46.63	35.00	11.63	PASS
Schwarz-D	No Fan	161.85	25.00	136.85	FAIL

5.4 Key Observations

1. Schwarz-D performs better than Gyroid in all cases (lower maximum temperatures)
2. Forced convection (fan) is absolutely necessary - all passive cooling tests failed
3. Best performance: Schwarz-D 1 CPU with fan (T_max = 46.63°C)
4. Without a fan, even the best design reaches 161°C (dangerous level)

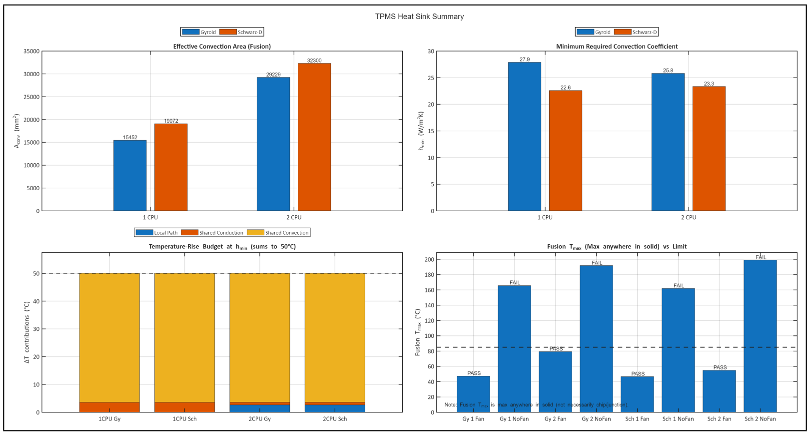


Figure 13: TPMS Heat Sink Summary - Comparison Charts

6 Cost Analysis

All heat sinks are manufactured using SLM (Selective Laser Melting) 3D printing with AlSi10Mg aluminum alloy.

6.1 Two CPU Configuration Costs

Table 5: Cost Analysis - 2 CPU Configuration

Cost Component	**Gyroid [USD]**	**Schwarz-D [USD]**
Material (ρ_Al = 100 USD/kg)	15.50	12.10
Machine Time (75 USD/hr)	147.75	116.25
Post-Processing	60.00	60.00
Heat Sink Cost	223.25	188.35
Fan + TIM + Assembly	30.00	30.00
TOTAL	**253.25**	**218.35**
Schwarz-D Savings	**34.90 USD (13.8%)**	

6.2 Single CPU Configuration Costs

Using a scaling factor of 0.583 from 2 CPU to 1 CPU designs:

Table 6: Cost Analysis - 1 CPU Configuration

Cost Component	**Gyroid [USD]**	**Schwarz-D [USD]**
Material	9.04	7.05
Machine Time	86.14	67.77
Post-Processing	60.00	60.00
Heat Sink Cost	155.18	134.82
Fan + TIM + Assembly	30.00	30.00
TOTAL	**185.18**	**164.82**
Schwarz-D Savings	**20.36 USD (11.0%)**	

6.3 TIM Cost Analysis

We selected Arctic MX-4 thermal interface material:

Table 7: Arctic MX-4 TIM Properties

Property	**Value**
Thermal Conductivity	8.5 W/mK
Density	2.50 g/cm^3
Price (4g tube)	$5.50 USD
Recommended Thickness	0.05 - 0.1 mm

TIM Volume Calculation (2 CPU):

- CPU side: $2 \times (10 \times 10) \times 0.1 = 20$ mm^3
- Heat sink side: $(60 \times 36) \times 0.1 = 216$ mm^3

- Total volume: 236 mm³ = 0.236 cm³

TIM Cost:

$$m_{TIM} = \rho \times V = 2.5 \times 0.236 = 0.59 \text{ g} \quad (25)$$

$$C_{TIM} = \frac{0.59}{4} \times 5.50 = 0.81 \text{ USD} \quad (26)$$

Table 8: TIM Cost Summary

Configuration	TIM Area [mm²]	TIM Mass [g]	Cost [USD]
2 CPU	2,360	0.59	0.81
1 CPU	1,000	0.25	0.34

Note: One 4g tube of Arctic MX-4 is enough for about 6-7 applications (2 CPU) or 16 applications (1 CPU).

7 Discussion

7.1 Thermal Performance

Our analysis clearly shows that:

1. **Schwarz-D is better than Gyroid:** In every test, Schwarz-D achieved lower maximum temperatures. For the 2 CPU case with fan, Schwarz-D reached only 54.81°C while Gyroid reached 76.37°C - a difference of 21.56°C.

2. **A fan is absolutely necessary:** Without forced cooling, all designs failed. Even the best design (Schwarz-D 1 CPU) reached 161.85°C without a fan, far above the 85°C limit.

3. **Higher surface area helps:** Schwarz-D has an A/V ratio of 2.089-2.111, compared to Gyroid's 1.484-1.706. This means more surface area for the same volume, leading to better heat transfer.

4. **Air velocity matters:** With a fan, air velocities reached 19-24 m/s. Without a fan, natural convection only produced 0.25-0.28 m/s - almost 100 times slower.

7.2 Cost Effectiveness

Schwarz-D is not only better thermally but also cheaper:

- 2 CPU: Saves $34.90 (13.8%)
- 1 CPU: Saves $20.36 (11.0%)

The lower cost comes from:

- Less material needed (lighter design)
- Less printing time (simpler geometry)
- Same post-processing cost

7.3 Design Trade-offs

While Schwarz-D performs better, we should consider:

1. **Structural strength:** Gyroid might be stronger due to its curved surfaces
2. **Print reliability:** Schwarz-D's diamond pattern might be easier to print
3. **Air flow resistance:** Both designs allow good air flow, but Schwarz-D channels are more direct
4. **Cleaning:** Both structures have complex internal channels that are hard to clean

7.4 Required Convection Coefficient

From our calculations, minimum required h values are:

Table 9: Minimum Required Convection Coefficients

Configuration	Gyroid h_min	Schwarz-D h_min
1 CPU (20W)	27.139 W/m²K	22.853 W/m²K
2 CPU (35W)	25.803 W/m²K	23.384 W/m²K

All these values are easily achievable with a small cooling fan. Natural convection provides only about 5-10 W/m²K, which is not enough.

8 Conclusions

Based on our complete thermal and cost analysis, we conclude:

1. **Best Design: Schwarz-D with forced cooling**
   - Lowest temperatures (46.63°C for 1 CPU, 54.81°C for 2 CPU)
   - Lower cost than Gyroid
   - Good safety margin (30-40°C below limit)
2. **Forced cooling is mandatory**
   - Natural convection cannot handle the heat load
   - All passive designs exceeded 160°C
   - A small fan makes a huge difference
3. **TPMS structures are effective**
   - High surface area enables good heat transfer
   - 3D printing allows complex geometries
   - Lighter than traditional finned heat sinks
4. **Design recommendations**

- Use Schwarz-D for best performance and cost
- Install at least a 25 W/m²K cooling fan
- Apply TIM correctly (0.1 mm thickness)
- Consider Gyroid only if structural strength is critical

8.1 Future Work

To improve this study further, we suggest:

1. Test other TPMS structures (Primitive, I-WP)
2. Optimize cell size and wall thickness
3. Test with different materials (copper, graphene-enhanced aluminum)
4. Build and test physical prototypes
5. Study long-term reliability and thermal cycling
6. Analyze noise levels from different fan speeds
7. Investigate hybrid designs (combining multiple TPMS)

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A Appendix A: Calculation Details

A.1 Thermal Resistance Formulas

General conduction resistance:

$$R_{cond} = \frac{L}{k \cdot A} \tag{27}$$

General convection resistance:

$$R_{conv} = \frac{1}{h \cdot A} \tag{28}$$

Series resistances:

$$R_{total} = R_1 + R_2 + R_3 + ... + R_n \tag{29}$$

Parallel resistances:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n} \tag{30}$$

A.2 Temperature Calculations

For single heat source:

$$T_j = T_\infty + Q \cdot R_{total} \tag{31}$$

For multiple heat sources:

$$T_{j,i} = T_\infty + Q_i \cdot R_{path,i} + Q_{total} \cdot R_{common} \tag{32}$$

B Appendix B: TPMS Mathematical Definitions

B.1 Gyroid

Implicit equation:

$$\sin(2\pi x/a)\cos(2\pi y/a) + \sin(2\pi y/a)\cos(2\pi z/a) + \sin(2\pi z/a)\cos(2\pi x/a) = t \tag{33}$$

where a is the cell size and t is the threshold value (typically 0).

B.2 Schwarz-D (Diamond)

Implicit equation:

$$\cos(2\pi x/a)\cos(2\pi y/a)\cos(2\pi z/a) - \sin(2\pi x/a)\sin(2\pi y/a)\sin(2\pi z/a) = t \tag{34}$$

C Appendix C: Material Selection Justification

C.1 Why AlSi10Mg for Heat Sinks?

- Good thermal conductivity (177 W/mK)
- Lightweight (2.7 g/cm^3)
- Excellent for SLM 3D printing
- Lower cost than copper
- Good corrosion resistance

C.2 Why Arctic MX-4 TIM?

- High thermal conductivity (8.5 W/mK)
- Non-conductive (electrically safe)
- Long lifespan (8+ years)
- No curing or burn-in time
- Easy to apply
- Good price/performance ratio

D Appendix D: Detailed Hand Calculations - Two CPU Resistance Corrected Version

This appendix presents the complete hand calculations for the two CPU configuration thermal analysis, showing the step-by-step methodology used to determine the required convection coefficient.

D.1 Thermal Resistance Network Diagram

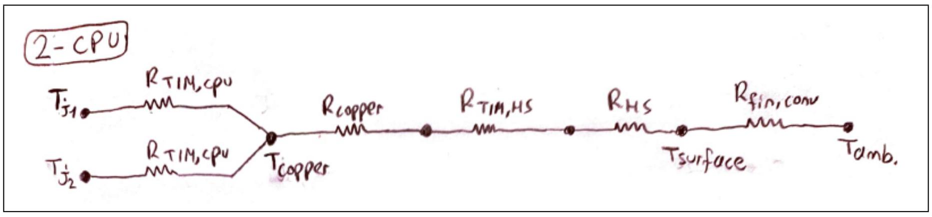

Figure 14: Two CPU Thermal Resistance Network - Hand Calculation Methodology

D.2 Methodology and Assumptions

Before the calculation, the engineering approach is defined as follows:

1. **Nodal Analysis:** Since two CPUs are independent heat sources ($Q_1 = 20W$, $Q_2 = 15W$), we calculate the temperature drop based on the physical junction of heat fluxes rather than using an electrical parallel equivalent resistance.

2. **Worst-case Design:** The design is based on the hotter component, CPU 1 (20W), ensuring it does not exceed the limit of 85°C.

3. **Neglecting R_{chip}:** Since no data for the chip was provided in the project brief and $R_{chip} \ll R_{TIM}$, the analysis starts from the junction-to-case (TIM interface) boundary.

4. **Neglecting Radiation:** Radiation effects are assumed to be negligible due to forced convection.

Note: The diagram above represents the correct physical model where heat fluxes start independently and merge at the copper spreader.

D.3 Given Parameters

$$Q_1 = 20W, \quad Q_2 = 15W, \quad Q_{total} = Q_1 + Q_2 = 35W$$
$$T_{j,max} = 85C, \quad T_{amb} = 35C$$

D.4 Step-by-Step Calculations

D.4.1 Thermal Interface Material - CPU Side

$$R_{TIM,CPU} = \frac{L_{TIM}}{k_{TIM} \cdot A_{TIM,CPU}} \tag{35}$$

Given:

- $L_{TIM} = 0.1 \times 10^{-3}$ m
- $k_{TIM} = 10$ W/(m·K)
- $A_{TIM,CPU} = 10^{-4}$ m²

$$R_{TIM,CPU} = \frac{10^{-4}}{(10\frac{W}{m \cdot K})(10^{-4}m^2)} = \frac{10^{-4}}{10 \times 10^{-4}} = 0.1\frac{C}{W} \tag{36}$$

D.4.2 Copper Heat Spreader

$$R_{copper} = \frac{L_{copper}}{k_{copper} \cdot A_{copper}} \tag{37}$$

Given:

- $L_{copper} = 3 \times 10^{-3}$ m
- $k_{copper} = 400$ W/(m·K)
- $A_{copper} = 60 \times 36 \times 10^{-6}$ m²

$$R_{copper} = \frac{3 \times 10^{-3}}{(400\frac{W}{m \cdot K})(60 \times 36 \times 10^{-6}m^2)} = 0.00347\frac{C}{W} \tag{38}$$

$$\boxed{R_{copper} = 0.00347\frac{C}{W}}$$

D.4.3 Thermal Interface Material - Heat Sink Side

$$R_{TIM,HS} = \frac{L_{TIM}}{k_{TIM} \cdot A_{TIM,HS}} \tag{39}$$

Given:

- $L_{TIM} = 0.1 \times 10^{-3}$ m
- $k_{TIM} = 10$ W/(m·K)
- $A_{TIM,HS} = 60 \times 36 \times 10^{-6}$ m²

$$R_{TIM,HS} = \frac{10^{-4}}{(10\frac{W}{m \cdot K})(60 \times 36 \times 10^{-6} m^2)} = 0.0046 \frac{C}{W} \tag{40}$$

$$\boxed{R_{TIM,HS} = 0.0046 \frac{C}{W}}$$

D.4.4 Heat Sink Base

$$R_{HS} = \frac{L_{HS}}{k_{HS} \cdot A_{HS}} \tag{41}$$

Given:

- $L_{HS} = 7 \times 10^{-3}$ m
- $k_{HS} = 177$ W/(m·K)
- $A_{HS} = 60 \times 36 \times 10^{-6}$ m²

$$R_{HS} = \frac{7 \times 10^{-3}}{(177\frac{W}{m \cdot K})(60 \times 36 \times 10^{-6} m^2)} = 0.0183 \frac{C}{W} \tag{42}$$

$$\boxed{R_{HS} = 0.0183 \frac{C}{W}}$$

D.4.5 Convection Resistance

$$R_{fin,conv} = \frac{1}{h_{conv} \cdot A_{conv}} \tag{43}$$

For Gyroid:

- $A_{conv,Gyroid} = 0.029 m^2$ (from Table 3)

$$R_{fin,conv,Gyroid} = \frac{1}{h_{conv}(0.029 m^2)} = \frac{34.48}{h_{conv}} \tag{44}$$

For Schwarz-D:

- $A_{conv,Schwarz} = 0.032 m^2$

$$R_{fin,conv,Schwarz} = \frac{1}{h_{conv}(0.032 m^2)} = \frac{31.25}{h_{conv}} \tag{45}$$

D.5 Temperature Analysis - Critical Path

D.5.1 STEP 1: Temperature Drop from Critical CPU to Copper Spreader

Starting from critical junction (85°C). Only 20W flows through this specific interface:

$$T_{copper} = T_{j,1} - (Q_1 \times R_{TIM,CPU}) = 85C - (20W \times 0.1 \frac{C}{W}) \tag{46}$$

$$\boxed{T_{copper} = 83C} \tag{47}$$

D.5.2 STEP 2: Temperature Drop from Copper to Heat Sink Surface

Here, heat fluxes merge. The total power is 35W:

$$T_{surface} = T_{copper} - (Q_{total} \times [R_{copper} + R_{TIM,HS} + R_{HS}]) \tag{48}$$

$$T_{surface} = 83C - (35W \times [0.00347 + 0.0046 + 0.0183]\frac{C}{W}) \tag{49}$$

$$T_{surface} = 83C - 0.924C \tag{50}$$

$$\boxed{T_{surface} = 82.076C} \tag{51}$$

D.5.3 STEP 3: Required Heat Transfer Coefficient

We must dissipate 35W to the ambient air (35°C) using the remaining temperature budget:

$$\Delta T_{conv} = T_{surface} - T_{amb} = 82.076C - 35C = 47.076C \tag{52}$$

$$\text{Required convection resistance: } R_{conv,req} = \frac{\Delta T_{conv}}{Q_{total}} = \frac{47.076C}{35W} = 1.345\frac{C}{W} \tag{53}$$

For Gyroid:

$$R_{fin,conv} = \frac{34.48}{h} = 1.345 \quad \Rightarrow \quad h = \frac{34.48}{1.345} = 25.64\frac{W}{m^2 \cdot K} \tag{54}$$

$$\boxed{h_{Gyroid,min} = 25.64\frac{W}{m^2 \cdot K}} \tag{55}$$

For Schwarz-D:

$$R_{fin,conv} = \frac{31.25}{h} = 1.345 \quad \Rightarrow \quad h = \frac{31.25}{1.345} = 23.23\frac{W}{m^2 \cdot K} \tag{56}$$

$$\boxed{h_{Schwarz,min} = 23.23\frac{W}{m^2 \cdot K}} \tag{57}$$

D.6 Conclusion

Minimum heat transfer coefficients required to keep the critical CPU below 85°C under full load:

- **Gyroid:** 25.64 W/m²·K
- **Schwarz-D:** 23.23 W/m²·K

Note: The calculation for Gyroid is shown in detail. For Schwarz-D, only A_{conv} changes (32,300 mm² vs 29,229 mm² for Gyroid), which results in a slightly lower required convection coefficient due to the larger surface area.